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# PITCAIRN FINANCIAL MANAGEMENT GROUP

September 15, 1989

CHAIRMAN'S OFFICE  
RECEIVED

Mr. David Ruder, Chairman  
Securities and Exchange Commission  
450 5th Street N. W.  
Washington, DC 20549

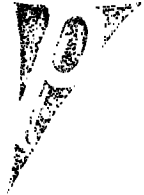
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SEC. & EXCH., COMM.

Dear Mr. Ruder:

I enjoyed your speech in Chicago on September 12 at The Northern Trust Conference.

Please find enclosed a paper articulating some of the systemic problems that have evolved within the capital markets. The current situation you had described is a function of the separation of ownership and control. I have been researching this problem for years and believe that we have to restrict the activities of money czars who impact the markets in non-economic fashion, and gamble with huge amounts of capital without regard to the underlying corporation, or the needs of the beneficial owners.

To use your example of capital market technology as the super highway, I would contend that the drivers of the automobiles do not own them, and therefore are not concerned with the consequences of their actions. Never before has so much capital been tossed around so mechanistically. This is a function of the passage of ERISA creating the situation where institutional money managers are rewarded in the near term for a transactions orientation. This transactions orientation has lead to increased expenditures on technology to increase transactions. This technology, the super highway, exceeds the ability of decision-makers to thoughtfully process it. The reaction is delegation to machines and mechanistic approaches. We must realign the interests of the owners and those in control.



Mr. David Ruder  
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September 15, 1989

I have enclosed a copy of my paper on Ownership and Control. It is an internal document for the billionaire family that employs me.

Sincerely,

*Mark Cunningham*

D. Mark Cunningham  
Vice President

/lck  
Enclosure

## OWNERSHIP AND CONTROL

The ownership of most large corporations today is represented by publicly traded corporate stock. Individual investors control some of these shares. Individual investors do not, however, hold most of the corporate shares. With the passage of the Employee Retirement Income Security Act in 1974 (E.R.I.S.A.), institutionally invested assets have grown dramatically. Today, institutional investors own most of the corporate stock in America and account for about 82% of the trading volume. Such investors are often viewed as abstractions: bank trust departments, insurance companies, mutual and pension funds. In reality, they are the individuals who manage these investments for their respective institutions.

Although an owner-entrepreneurial or an individual stockholder may have a considerable personal commitment to the welfare of a particular corporation, this is not the case with the institutional funds managers who dominate corporate stock trading today. They are dependent upon the overall performance and appearance of their investment portfolio, not on the respective fortunes of any individual company shares in their portfolio. The higher salaries, bonuses and career advancement of these institutional investors is dependent upon their regarding corporate stock in general as merely one commodity vehicle among many that can provide a return stream for the burgeoning assets of their clients. From this perspective, corporations with hundreds of thousands of employees are no different than purely financial instruments as Treasury bills, gold futures and Swiss francs.

Such institutional investors, unlike the neo-classic owner-entrepreneur, are passive, not active in the management of the corporation whose shares they hold. The only initiatives they can take are to buy more stock, if they like management's progress or to sell if they do not.

As those institutional investors do not own their assets but represent legal owners (trust beneficiaries, pension beneficiaries) who have even less involvement with the underlying companies and who are competed for by other institutional investors, the performance of their portfolios is evaluated in the short term usually from quarter to quarter. In this environment, it is not the manager's investable assets but their salaries that are at stake. If they are successful they are highly paid, if not, they are replaced. As a result, considerable pressure falls on them to keep step with the rest of the Wall Street herd. The

stronger the herd instinct, the less individual risk of loss of career.

To offset these considerable risks, institutional fund managers have developed a number of defensive tactics all of which involve shifting the attention of the beneficial owners from the resultant returns of investment decisions to transactions and processes which are ostensibly believed to increase return or reduce risk.

First, they assemble index funds that mechanically mimic the performance of the overall market. Market averages usually outperform most managers. Consequently index funds tend to do better than most conventional managers. This is because conventional managers can and do pick the wrong stock and incur high trading costs, paying for investment research and technology. Also, many fund managers switch out of out-of-favor stocks toward the end of a reporting period, a practice known as "window dressing".

Second, some funds hedge their holdings with puts, calls and futures, a way of minimizing volatility in the face of dramatic swings in the market. This gives them effective control over large amounts of assets over a period of time at a very low cost.

Third, some quantitative managers delegate to computers the decision-making process of when and what to buy with large blocks of stock. These program trades do not depend upon the fundamental performance or health of the underlying company at all. Instead, the computers relentlessly scour the tapes for exploitable gaps between current prices of stock index futures and the shares of the companies that comprise the indices.

The aim of all three tactics is to transfer the attention of the beneficial owner away from the returns to processing activity that can serve as an alibi, if necessary. Due to their focus on processing activity "models" and "tools" and "organizational discipline", which are all looking at pretty much the same things, these professional fund managers think more like speculators than investors. Additionally, because these professional fund managers have a fiduciary role to their trust and pension beneficiaries, they are often forced into accepting any advantageous market offer for the shares they are holding. Consequently, institutional investors are more likely to settle for short-term gains at the expense of long-term growth.

The surge in institutionally managed assets, their non-taxable nature, the competitive environment for more

assets, the fact that the institutional investor does not own the assets (is salary oriented as opposed to return oriented), the fiduciary liabilities and the focus on processing activity lead, in the aggregate, to a situation where institutional managed funds will ride with a currently favored company or industry, according to currently fashionable analysis, and then deftly leap to another purported growth opportunity at the first sign of trouble. A childish game of "chicken" is played as each competes to catch the peak market price before selling what all agree is a well managed company with good economic prospects. As a result, a healthy corporation has seen its stock fall, with all the entailing impact to its financials when short-term growth measures such as earnings estimates did not come through. This can be especially dramatic when a company's price earnings multiple is flying high based upon a good story and previous history of rapid growth.

One piece of bad news, however insignificant, and the hair-trigger fund managers stampede. Hundreds of million of dollars of market value disappear as the stock plummets low enough to attract new buyers. An announcement of Digital Equipment Corporation in 1983 that its first quarter earnings would be "substantially below" Wall Street expectations, caused a drop in IBM by three points, a loss in market value of \$1.8 billion, even though IBM had recently posted profit gains.

Further adding to the chaos are the arbitrageurs. Arbitrageurs have even shorter fuses than fund managers. They are not interested in the underlying company at all, but in small changes in the market value of its stock. Some of them harvest spreads for a few percentage points, the difference between the valuation of a stock and its options or between the price of a stock in one market and its price in another. Some live off differences between the price of the stock and the offer tendered by interested parties who want to take over the company. Still others exploit value differentials between the price of shares in the takeover candidate and in the company that is taking it over.

Arbitrageurs are a relatively new factor in the markets, having in a decade exploded in number from two dozen to 300 participants. They do not care about any company or its wealth building capability. They are pure speculators who might profit whether a firm is prospering, failing or standing still. They cycle huge amounts of money from one investment vehicle to another, figuring that a point here and a point there will add up to a good return over the course of a year.

To understand how the equity markets have come to such a state of separation of ownership and control, one has to consider the employers of these institutional investors - the pension plan sponsors, and the employers of the corporate pension plan sponsor - the corporate CEO, or more precisely, the executive committee. The Federal Pension Reform Act of 1974 requires that corporations contribute enough money to honor their pension commitments. A direct result was a massive in-house increase in the flow of money to manage pension funds, placing fund managers under more pressure to perform. Some estimates have indicated that a one percent improvement in the performance of corporate pension funds will result in a reduction of about 25% in the mandatory annual corporate contributions. Since pension fund contributions have been projected to be as large as 25% of corporate pretax profits, the urge to fund money managers who will provide above average performance is strong indeed.

Given such high stakes, many corporate CEO's view pension funds as independent profit centers and expect their managers to outperform the market and other money managers consistently. Because corporations divide their operations neatly into quarterly and yearly periods, money managers are expected to turn in above-average performances smoothly and on schedule. Good results should be almost as systematic as the flow of widgets down an assembly line. Consequently, those pension plan sponsors who perform are rewarded with six figure incomes; those that do not are replaced. Scrambling for higher returns on their investments, fund managers have channeled huge amounts of pension money into common stocks. Pension assets which should be prudent, long-term investments have been sunk into short-term and increasingly speculative vehicles.

Consequently pension plans through both secondary and tertiary financial instruments, such as index funds, have come to be the major holders of American corporate stocks; and corporate employees have in the aggregate become the predominant beneficial owners of American corporations leading to pension fund socialism. This ownership is so divorced from control that these individuals have not the least bit of effective control over any of the corporations of which they are part owners. Yet, all the workers share in the fortunes of corporate performance for better or worse; those who have control of the assets are not owners and those who own the assets have no control. Their respective motivations are not the same.

The pressure on near-term performance, at the expense of long-term wealth building, causes many financial officers through their pension fund consultants to monitor in minute

detail the performance of the professional investors they have hired (as well as those of promising replacements). Usually detailed review of the account takes place every three months through the exhaustive and exotic quantitative assistance of the manager's consultant. the institutional investor who moves out of step, resulting in returns below par for even a relatively short term, is under increasing pressure to perform. the "twelve/twenty-four rule" as it is called, is followed by not so few corporations. The manager who is 12% below the Standard & Poor's 500 for twenty-four months is replaced. In short, money managers have increasingly found themselves in an impossible position of demanding results far too quickly with an inevitable reliance on playing the popular trends.

A final source of pressure on the pension fund manager is the requirement of the Employer Retirement Income Security Act of 1974, which mandates that pension funds be "prudent", and tethers the executive committee of the corporation with a non-delegable liability. Not to be prudent can result in litigation against the money manager. Presently, the definition of what is prudent is extremely tenuous, which results in advisors acting as a herd. Those who take a unique stand that proves to be wrong may be staring at bankruptcy.

A final pressure towards consensus is the executive committee of the corporation. Although pension fund officers are responsible to their board of directors, they may be far more sophisticated in investment understanding than its members. The pension plan sponsor either says what the board wants to hear or they will be replaced.

These various pressures on near-term performance at the expense of long-term wealth building, combined with absentee-ownership of the assets, lead to an increasing interest in the ultimate speculative venture, market timing. Market timing is seductive. Over a ten year period, catching the major surges in the equity markets will triple the results of simply buying and holding the Standard & Poor's 500. Furthermore, market timing can be counted upon to deliver the activity upon which a hungry and transactions-based financial services industry feeds. Unfortunately again, the interests of the beneficial owners and those who invest the assets for them are again at odds. While market timing is clearly good for the professional investment community, providing fees through transactions, technical analysis, and consulting services, as well as alibis for poor performance, and providing continuous opportunities to succeed in the future, it has not been proven to benefit the beneficial owners who receive the

ensuing return stream.

William Sharpe, a Stanford University finance professor and well-published academician, determined that a money manager who wishes to market-time profitably must be right three out of four times after commissions and advisory costs. Yet another major study by Merrill Lynch concluded that the great majority of funds lose money as a result of their timing efforts, and when the effects of commission costs are included, no one succeeds.

Academic scrutiny has proved to be equally unsparing of the fundamental practitioner's abilities as it has of the technician's. It has been determined that if the degree of risk remains unchanged, there is no correlation between a fund's performance in one period and its performance in another. This analysis dispels the myth of the hot money manager. Funds in the top 10% in one period might be in the bottom 10% of the next or vice versa.

Nor has any link been found between portfolio turnover and subsequent performance. Rapid turnover does not improve results. Also, if the risk factor is held constant, there is little difference in the results of funds of various sizes.

A survey of 571 of the largest pension and profit sharing funds in the country managed primarily by banks and insurance companies for the three, five, and ten and fifteen year periods ended in 1978, indicated that only 22% did as well as the market. Another study covered 214 pooled equity funds, large banks, and insurance companies that managed over \$100 billion. The measurement was for one, three, five and ten year periods ending December 31, 1980. In every holding period, they examined banks pooled funds performance for 1962-1975 and found that 87% underperformed the Standard & Poor's 500. A Becker study through the end of 1981 found that the median of 3,500 of the largest profit sharing endowment and other tax-exempt funds with stockholdings totaling over \$125 billion did 20% worse than the Standard & Poor's 500 for the last fifteen years, and did 30% worse in the last decade. The unequivocal conclusion is second guessing corporate management is not rewarded. Consequently, the solution has to lie in working with management to closely align the interests of ownership and control.

In theory, management's interests are the same as those of the stockholders for whom they work. In times past, that theory made more sense than it generally does today. For example, during the 1890's, John Pitcairn served as Chairman of the Pittsburgh Plate Glass Company and also owned 50% of the capital stock. This focused his priorities clearly on



wealth-building. His performance as a manager was perfectly aligned with his interests as owner/entrepreneur. Annual returns on investment were superior, and today his family controls assets exceeding \$1 billion due to his success as a long-term wealth-builder.

However, the conceptualization of the modern business corporation was still evolving during the time of John Pitcairn. During the ensuing one hundred odd years, organizational structures have evolved from the few layers of management necessary for coping with the complexities of operating a nineteenth century glass company to the modern vertically integrated multinational corporation that is PPG Industries. The management team is largely a post World War II phenomenon, although it had been developing throughout the early decades of the twentieth century. The evolution of the professional management team created an organized cadre that had different motivations from the beneficial owners. As long as the owners were organized and ownership was concentrated this did not present much of a problem. However, The Securities Act of 1933 and The Securities Exchange Act of 1934, by increasing information dissemination to the investing public, mandated that executives should increase the numbers of public investors instead of maintaining their allegiance to the original entrepreneurs. Such actions legitimized the separation of ownership and control. By the early 1960's, individuals or families held the majority of stock in only five of the two hundred largest non-financial corporations in the nation, and a full 169 of these 200 companies were controlled by professional managers. As ownership of corporations became progressively more dispersed and diffused among thousands of passive stockholders, professional managers gradually assumed effective control.

These managers, like the institutional fund manager, have as primary rewards salaries, bonuses and career advancements within the company or the greater industry.

The CEO's legal responsibility is to serve the interests of the stockholders. The problem begins with the recognition that stockholders can have adverse interests and differing time horizons. Some own the stock for short-term returns, whether in the form of dividends or appreciation of the market value of the shares. Yet, these two forms of short-term return often require opposing business strategies. Long-term wealth-building requires investments in modernized production facilities, market development, new technologies or basic research; the money for which may have to come at the expense of short-term returns. Should ambitious long-term programs result in an undervalued share price, the company may become vulnerable as a takeover target. Once a

raider puts the company in play, senior management jobs are at risk.

In the face of such conflicting interests among the "owners", and often no interest at all by the trust and pension beneficiaries, who ultimately own the assets, management can become elites accountable only to themselves. If the executives' ultimate ambitions lie outside the corporation, then a spectacular short-term gain in sales or market share may be the strategy, especially if the financial press takes notice. Senior management seeks growth in whatever form suits their own purposes, and there can be trade-offs between corporate strategy and career strategy. When push comes to shove, as often is witnessed in hostile tenders, management tends to protect their own jobs rather than make personal sacrifices on behalf of ownership interests.

In theory, the board of directors should check management in their pursuit of self-interests. Yet, primarily because of the vagaries of the proxy process for shareholder voting, most directors are more closely affiliated with incumbent management than with the owners. Many are outside directors, others have significant ties to the enterprise other than ownership, and in most instances directors are more beholden to incumbent management than to the shareholders for their seat on the board. Consequently, while the board of directors may assist management in running the company, it is seldom an independent voice for shareholders.

The pressure on corporate management to keep the stock price up in the near term cuts off many long term wealth building opportunities. Because managements' personal interest are not aligned with those of the owners who are themselves ranging from disloyal to disinterested, and diverse in time horizon, pressure is placed upon short term performance to avoid a hostile takeover attempt. In the face of this near-term pressure often the only short-term solution to a problem is purely financial. Corporate management follows the paradigm of the institutional investor and acts as portfolio managers. The crux of the problem with the portfolio manager approach to corporate finance is the professional corporate managers who direct the affairs of absentee-owned companies tend to see their role and purpose to be "managing assets" rather than building profitable companies that increase their market share over time. Because of this orientation they focus their attention and energies on making short-term gains by managing their collection of companies as if they were a stock portfolio. They use debt to spur company growth through mergers and acquisitions and they improve corporate performance in the short run by stock buybacks and the restructuring of assets.

For such managers the financial transaction has become the mechanism for achieving professional success rather than technical knowledge of their industry. This type of manager is characterized by sophisticated financial and administrative skills, and a focus on getting quick results and immediate rewards. It is this combination that can prove so crippling to the long-term success of a company and its industry.

The epitome of the corporate manager/portfolio manager is the management of a conglomerate. New companies are acquired predominantly as investment and the financial return supplied by an acquired company is its only measure of value and success. Management strategies are variations on manipulations of assets. The objective is not to gain market share and build wealth but to manipulate assets to maximize their present value.

Michael Porter found in a study of diversification strategies at 33 major firms during the years between 1950 and 1986 that some 74 percent of all acquisitions made by these companies into unrelated industries were subsequently divested or closed down. In related fields the failure rate was still 50%.

In summary the current ownership structure of the corporation leads to a tug-of-war between various factions competing with conflicting personal motivations to extract as much personal wealth in the form of salaries, bonuses, career advancement, management fees, transactions charges and consulting fees from the legal owners of the corporation. This has happened because the beneficial owners are disorganized and corporate management and institutional investors are not. As ownership and control are further separated, the competing factions, the various owners and corporate management, are forced into a near-term focus on results which favors financial solutions at the expense of long-term growth. The pressure of substantial institutional shareholders upon management, like the pressure of venture capitalists on a new firm, forces managers and boards of directors to rivet their attention to short-run profits and the company's share prices.

Ownership and control when tightly linked with regard to interests boosts long-term returns, and permits the management more flexibility with regard to wealth building.

The most obvious method of reconstructing ownership and control in a large loosely held corporation is the leveraged buy-out. In a leveraged buy-out, a group of investors borrow the money, often by issuing high-interest weakly collateralized junk bonds, to purchase a company's

outstanding stock and thus, bring it once again under the control of the owner-entrepreneur. The business has a more focused set of objectives than one owned by thousands of passive pension and trust beneficiaries. While LBO's occur when management fails to run the business in the interests of the owners and the market value of a company falls below its maximum attainable value, LBO's incur so much debt that they often must liquidate assets to service the debt and while the owners are significantly enriched in the near-term often the corporation has entirely mortgaged its future for the present.

Buying a minority investment in a company differs fundamentally from buying control. With a minority interest the investor is a passive observer who only shares in what management decided to share with them. With control they are able to restructure the corporation. The two situations are so different that the shares are actually evacuated in two distinct markets. The secondary market trades claims on future dividends and price/earnings multiplies. The primary market of the corporate raiders trades control, and sells at a significant premium - the control premium. Because shares trading in the two markets are really different assets, they naturally sell at different prices. The minority investor receives the present value of cash flows to equity given current control. The raider has calculated a maximum premium over the minority share value that they will pay to gain control. It is also the expected increase in shareholder value created by the change in control. If a corporate raider pays fair market value of the minority shareholder for the target company all the increased value will be realized by all existing shareholders. At any lower price the remaining value goes in the raiders pocket.

This remaining value is derived from their sources; 1) tax shields, 2) incentive effects; and 3) controlling free cash flow. Tax shields consist of the interest expense and interest depreciation from debt financing. Incentive effects consist of increasing management's ownership of the company. Gaining control of free cash flow enables maximizing free cash flow in the interest of the owners and thereby increasing shareholder value. To the extent shareholders succeed in forcing management to increase value, the economy's resources are allocated more efficiently. In the restructuring following the LBO, management has probably invested much of their own resources in equity of the restructured company. Consequently management's own well-being is tied closely to that of the business. Moreover the huge debt service burden that restructuring frequently creates, forces management to generate healthy cash flows or face bankruptcy.

Many hostile takeovers occur in mature or declining industries where there are low numbers of investment opportunities, and business often have large free cash flow. Industry decline creates real concern on the part of executives regarding the survival of their organization. Although the proper strategy from a business perspective may be to shrink or liquidate the business, management may refuse to do so. Out of a commitment to the business, the employees, the community and their own personal welfare, management may continue to reinvest in the business despite poor returns. The objective of the hostile takeover is to wrest control of free cash flow from current management and put it in the hands of the rightful owners.

If management wishes to avoid a corporate raider, there are two strategies. First look at the company as both a business and an investment. Management should work to increase free cash flow and avoid uses of free cash flow that reduce firm value. They should take wealth-building seriously. Second, management should work to ensure that their board of directors really represent the owners' interest. These strategies will reduce the disparities between ownership and control.

Separation of ownership and control in a corporation hinders long term performance pressure on all involved, leading to financial solutions to all business problems and a view by corporate management that they are managing assets. Managements autonomous position and loose relationship with the diverse owners lead them into conflicts between corporate and personal career strategy which are manifest in a tug-of-war over the wealth streams that is generated from the corporation, and which can be diverted to dividends, retain earnings or bonuses for management. When ownership and control become too disparate and cash flow is diverted from the owners the corporation struggles inefficiently, market price suffers, and like a thrashing swimmer attracting sharks, the corporate raiders redress this inefficiency in dramatic fashion.

The key to longterm wealth building is to avoid such inefficiencies. The Pitcairn Family Office has looked to history to determine this optimal long term strategy for wealthbuilding in conducting this research we have had the opportunity of interviewing and sharing information with thirty-six other families in this country who have had similar experiences to the Pitcairn family with PPG Industries. These families have been quite successful and have in the aggregate amassed wealth exceeding \$29 billion. Interviewing these families we determined that they had built

their wealth in many different industries. They did have however, certain common experiences. First they had not amassed the wealth over a short period of time. As a group they had accumulated the wealth over long periods of time, more than 20 years on average, and had during that time enjoyed superior rates of return, returns that would have been in the top quartile annually of institutional investors. None of the families had amassed the wealth as consumers of institutional investors although two of them were in financial services and had amassed the wealth by managing other peoples' money for them.

To determine the plausibility of any family amassing such wealth as a consumer of return streams from institutional investors we examined the return streams of 1,120 institutional investment managers over an 8 year period, to reach beyond the typical 5-7 year equity market cycle.

#### Analysis of 1,120 Institutional Investment Managers

<u>Time Period</u>	<u>Incidence of Success (Top Quartile)</u>	<u>Stock Funds</u>	<u>Bond Funds</u>	<u>Balanced Funds</u>
8 years	8 successes	0.2%	0.0%	0.2%
8 years	>6 successes	1.8%	0.0%	1.4%
8 years	>4 successes	14.7%	11.9%	13.0%
8 years	>2 successes	50.3%	52.3%	51.0%
8 years	>0 successes	100.0%	100.0%	100.0%

We then questioned what the results would look like should the odds of being in the top quartile be purely a matter of luck i.e. statistically independent. To do this we used the binomial probability formula which for 8 consecutive successes with a 25% chance of being in the top quartile would be:

$$\frac{8!}{8!0!} (0.25)^8 (0.75)^0 \text{ or } 0.00002 \text{ or } 0.0\%$$

Continuing this analysis the results were:

<u>Time Period Observed</u>	<u>Pure Luck Incidence of Success</u>	<u>Anticipated Percent of Money Managers</u>
8 years	8 successes	0.0%
8 years	>6 successes	0.4%
8 years	>4 successes	11.4%
8 years	>2 successes	63.3%
8 years	>0 successes	100.0%

These "pure luck" results were close enough to the observed results that we derived the implied probability of the observed results through the binomial formula.

#### Analysis of 1,120 Institutional Investment Managers

<u>Time Period Observed</u>	<u>Incidence of Success (Top Quartile)</u>	<u>Stock Funds</u>	<u>Bond Funds</u>	<u>Balanced Funds</u>
8 years	8 successes	45%	25%	45%
8 years	6 successes	32%	25%	31%
8 years	4 successes	27%	25%	26%
8 years	2 successes	20%	21%	21%
8 years	0 successes	25%	25%	25%

The results in all cases but particularly in the bond fund, were close enough to 25% as to indicate asset returns are independent. The influence of the equity component of the balanced funds is striking due to the apparent randomness of fixed income returns.

Given these results the odds of selecting an institutional investor that would deliver consecutive top growth results over an 8 year period is about 2 in 1,000 or put another way, the likelihood of failing to select one is 99.8%. However, the 36 families examined had actually enjoyed the equivalent of top quartile returns for more than a 20 year period at some point. It is important to point out that one could have an 8 year cumulative return equivalent to having been in the top quartile for 8 consecutive years through many permutations beyond this analysis. This analysis does not argue that there are no superior equity managers it simply argues that in fact annual returns of institutional investment managers in the aggregate are independent a contention supported by many other research studies and which has caused the creation of the manager consultant industry.

Since the aggregate institutional investment returns are independent then we can examine the likelihood of consuming institutional investor returns and matching the long term performance of the thirty-six high net worth families examined.

It is obvious that the likelihood of duplicating their performance as consumers of institutional investors is extremely remote.

All of the families had in fact built their wealth through the corporate structure, which regardless of industry had in

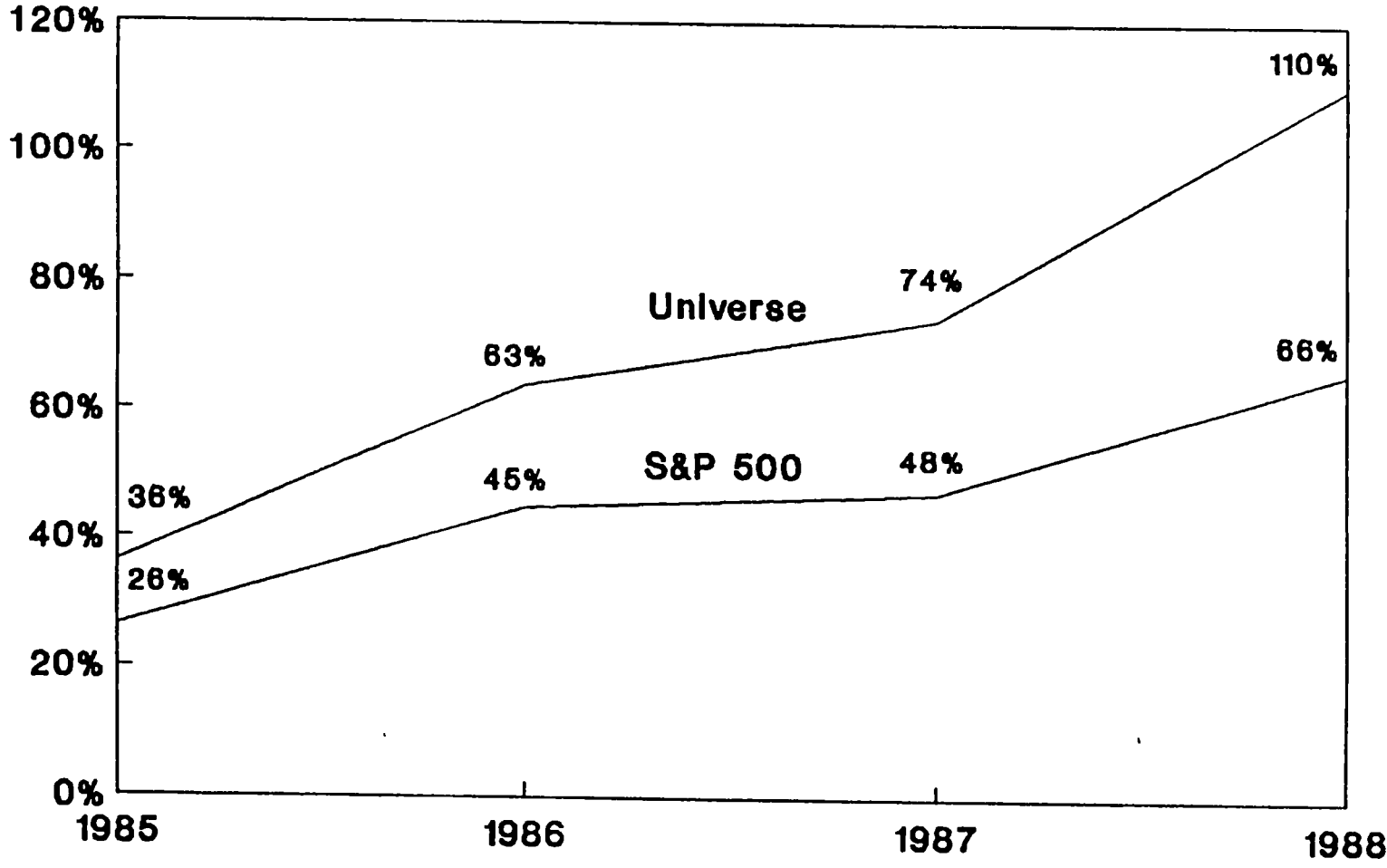
place a management team that had enjoyed much success as an economic engine.

Believing that a tight linkage between ownership and control is desirable, we examined the results of companies where a family owned significant blocks of the outstanding equity.

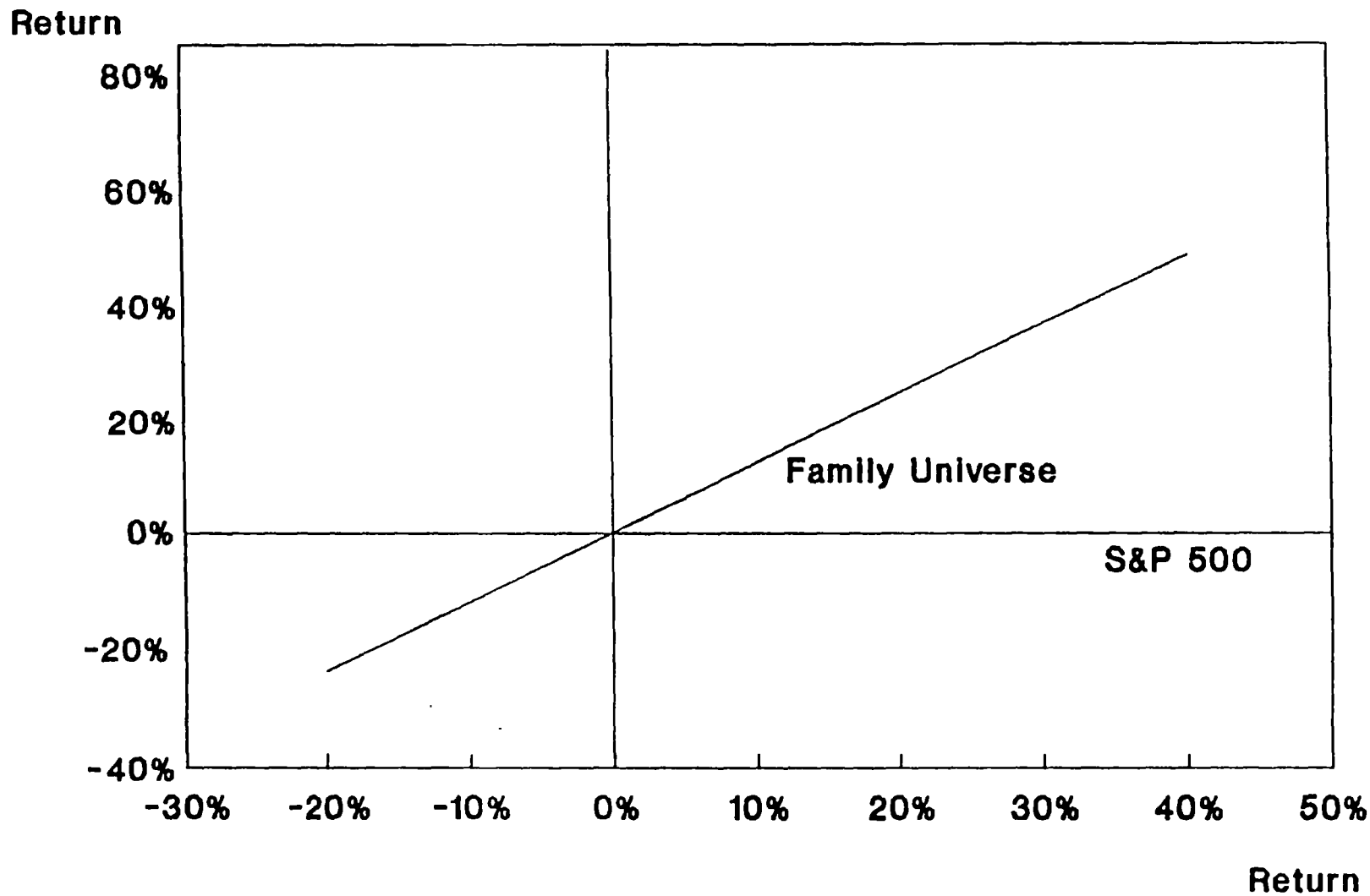
An analysis of 2000 companies yielded approximately 300 companies where families held more than 10% of the outstanding equity. Specifically the percentage ownership ranged from 10% to 54%. This aggregation equal-weighted was labeled the Family Universe. When compared to the S&P<sub>500</sub> over the years 1985 through 1988, it significantly outperformed the S&P<sub>500</sub>.



# CUMULATIVE RETURNS: S&P 500, FAMILY UNIVERSE

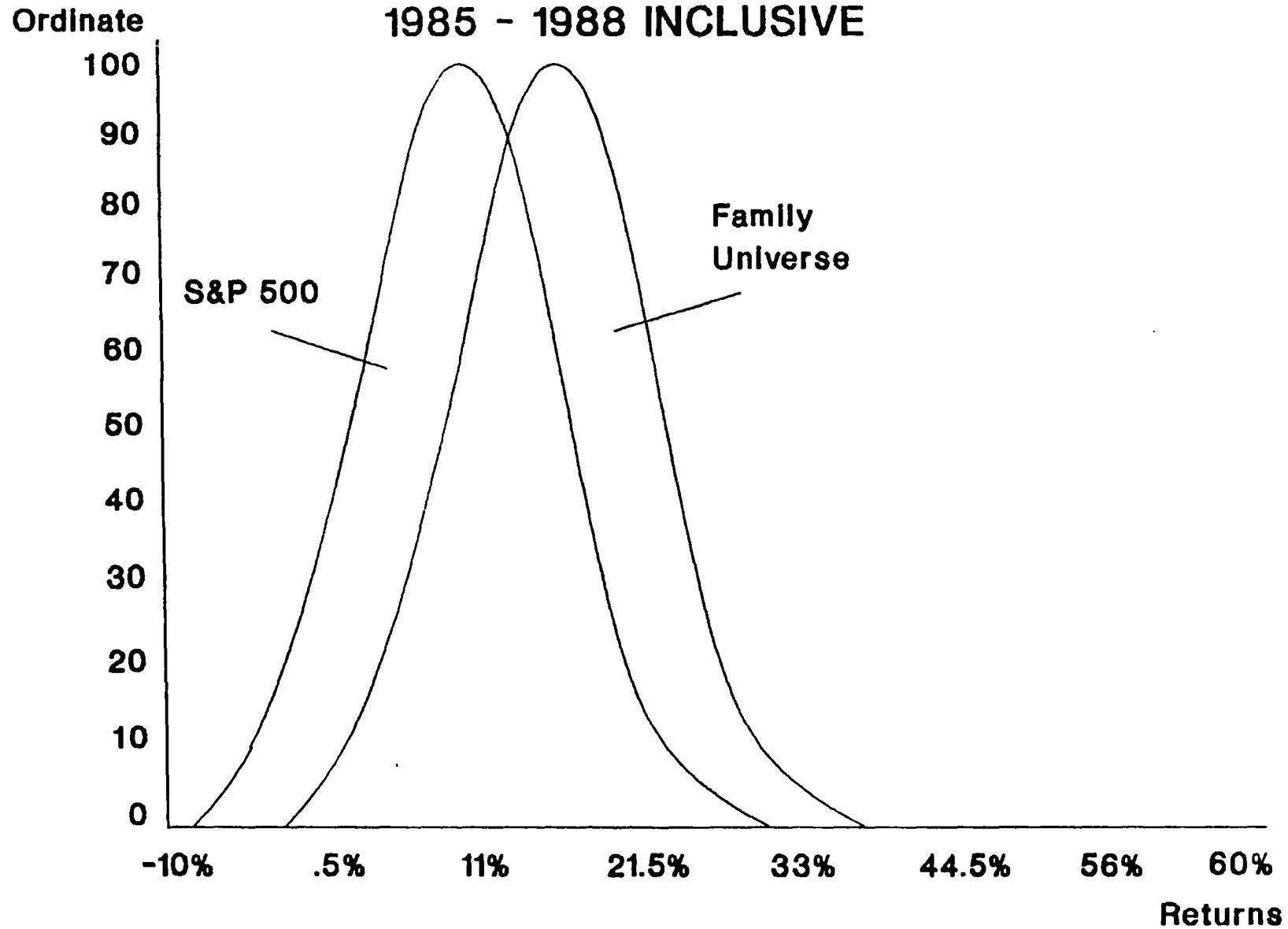


# LINEAR REGRESSION: S&P 500, FAMILY UNIVERSE



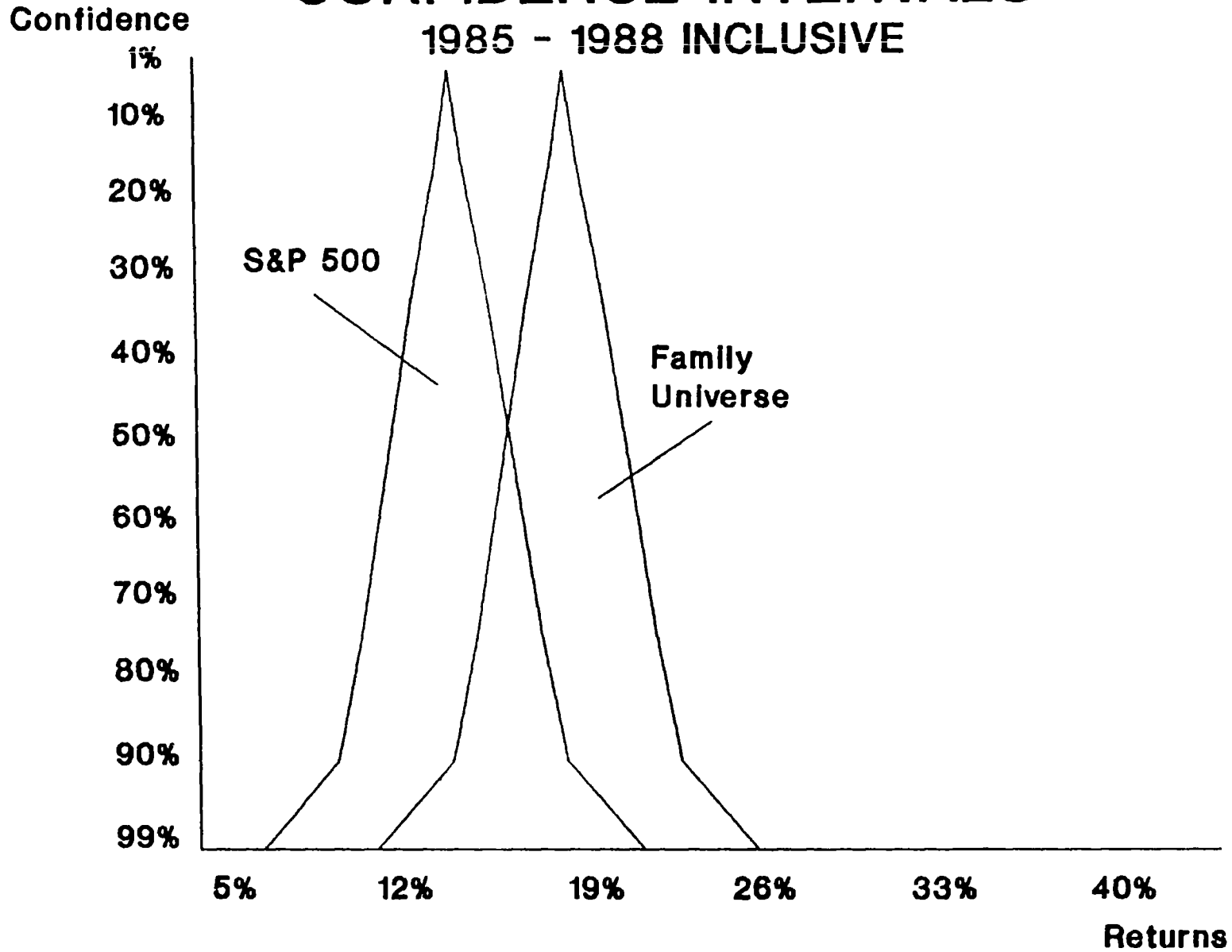
# NORMAL DISTRIBUTIONS

1985 - 1988 INCLUSIVE



# CONFIDENCE INTERVALS

1985 - 1988 INCLUSIVE



Further analysis then looked at these same companies over the 20 year period 1968 - 1988.

The results demonstrate substantial outperformance of the S&P<sub>500</sub> over time and in almost every year.

FAMILY UNIVERSE

ANNUAL RETURNS

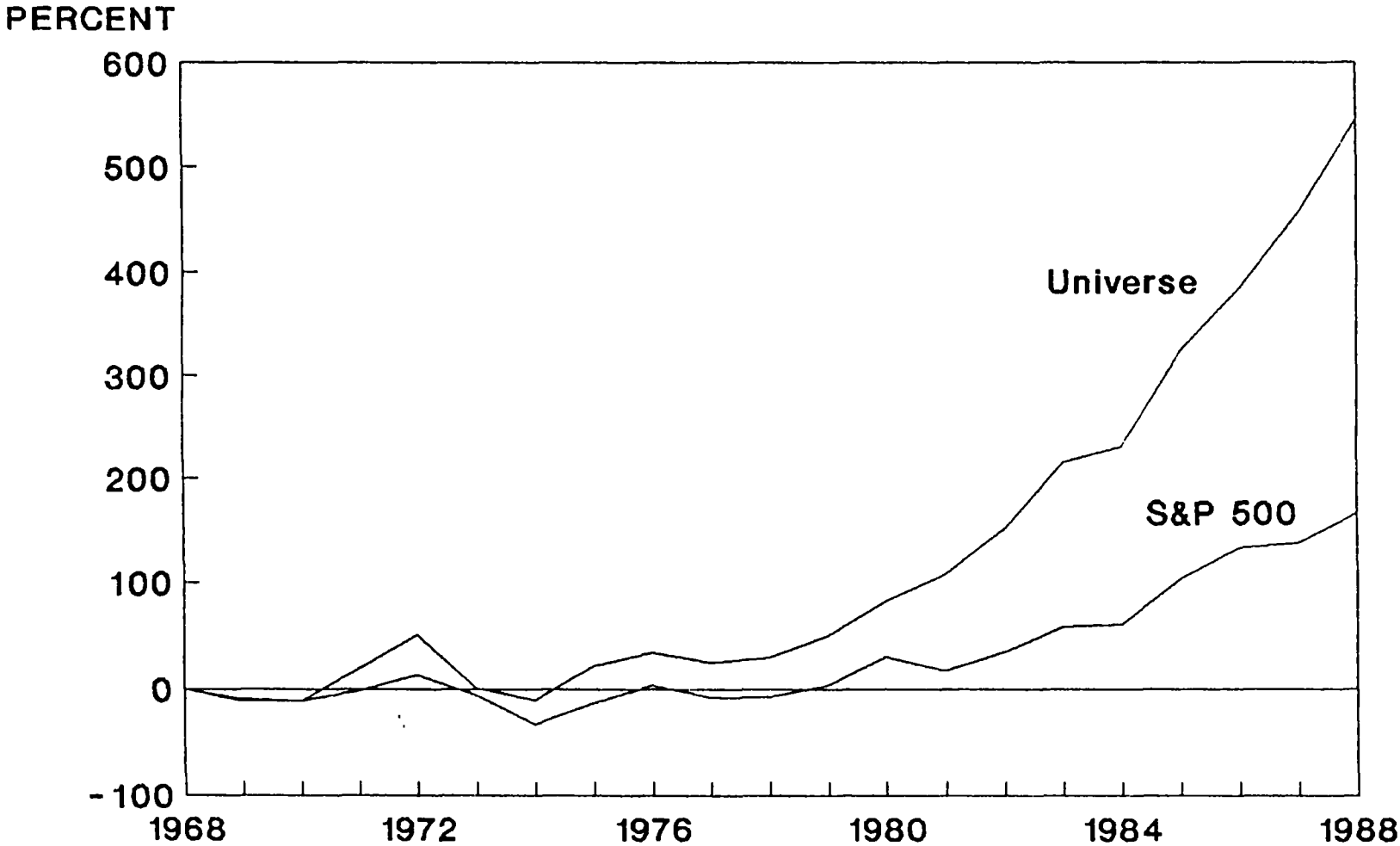
1969	1970	1971	1972	1973	1974	1975	1976	1977	1978	1979	1980	1981	1982	1983	1984	1985	1986	1987	1988	YEAR
-7.42056	-2.89506	7.287118	11.00436	2.948644	-1.90253	4.629994	8.122367	7.875955	9.050928	10.04460	11.73227	11.56259	13.70508	15.04022	14.43194	15.78246	16.22696	16.22003	16.67704	1969
	1.872639	15.49524	17.92754	5.717567	-0.76007	6.785712	10.54631	9.957769	11.05318	11.96290	13.65868	13.31018	15.51714	16.83902	16.06006	17.41222	17.79265	17.69771	18.10637	1970
		30.93949	26.88927	7.031191	-1.40755	7.796402	12.06208	11.16402	12.25745	13.14401	14.90982	14.41157	16.73350	18.07751	17.14399	18.52875	18.86651	18.70168	19.08056	1971
			22.94688	-3.23234	-10.3050	2.680427	8.626635	8.171584	9.815696	11.09683	13.25454	12.87815	15.52112	17.04450	16.14720	17.68866	18.10241	17.97595	18.41742	1972
				-23.8372	-23.3886	-3.30343	5.315192	5.436825	7.767771	9.499897	12.09801	11.81159	14.80367	16.54390	15.59783	17.29362	17.76377	17.65179	18.13994	1973
					-22.9374	8.954487	17.32945	14.36804	15.51492	16.33012	18.46149	17.30859	20.15925	21.60858	20.06682	21.59117	21.77856	21.36354	21.64856	1974
						54.04472	44.77343	30.45431	27.81645	26.31783	27.26222	24.56593	27.01981	27.93155	25.51076	25.73815	26.51192	25.67848	25.68072	1975
							36.06015	20.05057	20.10656	20.20311	22.49265	20.23328	23.56730	24.99540	22.68625	24.28921	24.26744	23.56499	23.72867	1976
								5.924767	12.84560	15.33929	19.31767	17.29607	21.59964	23.48994	21.10972	23.04586	23.14591	22.48762	22.75296	1977
									20.21863	20.35584	24.14823	20.32474	25.00266	26.68852	23.44987	25.37204	25.22445	24.28012	24.40943	1978
										20.49321	26.16094	20.36013	26.22813	28.02370	23.99679	26.12608	25.86466	24.73979	24.83646	1979
											32.09528	20.29365	28.19980	29.97874	24.70964	27.09015	26.65131	25.28103	25.32861	1980
												9.546409	26.29535	29.28080	22.92868	26.11212	25.76606	24.33669	24.50752	1981
													45.60510	40.44360	27.74333	30.63107	29.28749	26.98902	26.80349	1982
														35.46507	19.65176	25.99011	25.50224	23.56176	23.91721	1983
															5.684387	21.50412	22.34687	20.75310	21.72857	1984
																39.69189	31.63879	26.23916	26.10663	1985
																	24.04995	20.00672	21.87843	1986
																		16.09527	20.80696	1987
																			25.70988	1988

FAMILY UNIVERSE - SLP500

DIFF  
ANNUAL  
RETURNS

1969	1970	1971	1972	1973	1974	1975	1976	1977	1978	1979	1980	1981	1982	1983	1984	1985	1986	1987	1988	YEAR
0.744272	-0.49486	4.429164	4.407232	0.953713	1.376372	2.961080	3.975024	5.066822	5.875636	5.608184	5.284877	6.049165	7.200001	7.558437	7.058202	7.145321	7.067877	7.260454	7.355122	1969
	-1.87491	6.639259	5.900110	1.014274	1.511130	3.378369	4.510201	5.687731	6.534429	6.174942	5.727792	6.568848	7.790979	8.142596	7.561487	7.628254	7.518267	7.698197	7.776949	1970
		16.72354	10.46830	2.007367	2.312972	4.456967	5.639673	6.819130	7.641899	7.126870	6.601118	7.393835	8.668979	8.990760	8.299856	8.330083	8.170819	8.322887	8.373473	1971
			4.296677	-3.94136	-1.25505	1.894264	3.697912	5.386833	6.504102	6.062267	5.583049	6.554797	7.999120	8.394645	7.703509	7.771480	7.637510	7.832659	7.913345	1972
				-9.31793	-3.01736	1.246027	3.561341	5.560945	6.812464	6.278452	5.725409	6.776270	8.335397	8.738616	7.963964	8.020963	7.861452	8.053441	8.126010	1973
					2.885213	0.091122	9.489827	10.52946	11.14356	9.812804	8.713718	9.533411	11.06172	11.27250	10.15429	10.05943	9.731071	9.802201	9.768435	1974
						16.89477	14.74691	14.29486	14.14305	11.80529	10.11019	10.88256	12.53236	12.61864	11.19009	10.99349	10.54618	10.55928	10.46767	1975
							12.78706	13.14902	13.32902	10.74105	8.975722	10.05047	11.99600	12.15536	10.65501	10.49203	10.05706	10.11343	10.04998	1976
								13.22050	13.46863	10.12807	8.116673	9.559544	11.86768	12.06645	10.40963	10.25554	9.804326	9.889161	9.839268	1977
									13.68866	8.271949	5.995155	8.463722	11.50684	11.79657	9.908308	9.782559	9.323207	9.471176	9.453149	1978
										2.545838	1.729062	6.663457	10.92081	11.38217	9.242559	9.180939	8.735471	8.922119	9.001731	1979
											0.800116	8.655060	13.75283	13.65648	10.57971	10.30793	9.635709	9.780525	9.724088	1980
												14.62154	19.44349	17.55995	12.72724	12.83400	10.97458	10.93181	10.72761	1981
													25.32766	19.24118	11.92170	11.18888	10.04924	10.17175	10.04219	1982
														13.33054	5.995741	6.824993	6.522391	7.424487	7.729539	1983
															-0.08169	3.796526	4.400365	6.068545	6.695051	1984
																8.694524	7.085729	8.417493	8.632056	1985
																	5.624165	8.267566	8.593796	1986
																		10.66520	10.00839	1987
																			9.269866	1988

# CUMULATIVE RETURNS: S&P 500, FAMILY UNIVERSE





Further analysis indicates concentration of shares in a family's hands, the union of control and ownership leads to 1) a strong sense of mission; 2) well-defined long term goals; 3) a capacity for self-analysis; 4) the ability to bring out the best in employees including management, 5) the foresight to anticipate and adapt to major changes without losing momentum, 6) a lessening of buracracy and 7) a lessening of management politics.

Our research indicates that family ownership boosts motivation only where certain conditions are evident; 1) there are tangible financial rewards for employees resulting from ownership; 2) there is ongoing communication between management and the family and with clear management accountability; 3) there is effective planning and structure.

Still other factors must be avoided which are unique to family controlled businesses. Family businesses' strength can also be their Achilles heel. Such businesses sometimes come apart because of disagreement between family members that may have nothing to do with the business.

A pathological problem to avoid in family controlled companies is the founder's trap syndrome in which the founder excessively dominates the organization and the organization's success is almost exclusively dependent upon the founder's availability. The founder is the biggest asset and liability the company has. Frequently when this person exits, the company dies or the family that owns it loses control entirely. This usually happens within three generations. The trap is that the company cannot extract itself, by itself, from this predicament.

Still if these challenges are overcome the most consistently successful of all enterprises seem to be those that are family managed. The executives have a commitment that is larger than that of the business, it is to the other members of the family both present and future.

Every decision is based on what is best for the family. As a result it becomes possible to transcend areas of economic activity and move into those areas which hold more potential. The Family and their employees are more important than the product or service to that company. The company does not become trapped in one field or endeavor. Our research indicates that becoming consistently successful seems to combine the concern for long-range success that is found in family organizations with the efficiency of the professionally managed corporation.

Other key criteria that distinguish the better Family

controlled companies for investment purposes:

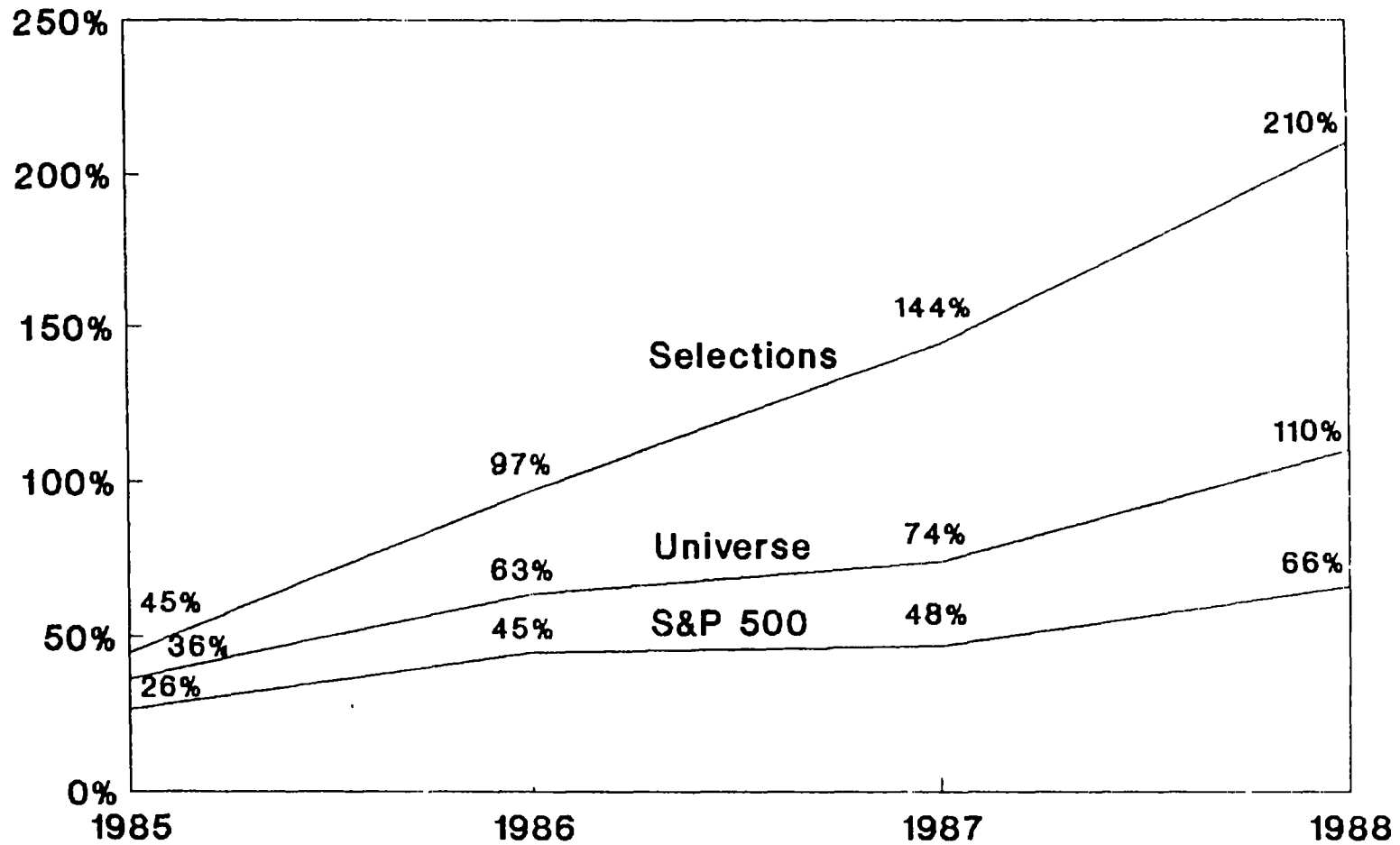
1. They develop a strategic focus
2. Pay close attention to management transition
3. Carefully select and socialize their employees
4. Emphasize long-term rewards
5. Create an effective organizational design
6. Grow where the business is, do not try to make a market where none exists.
7. Do not assume a good manager can run anything (Portfolio manager approach)
8. Do not load the producing personnel down with administrative functions (Function over form.)
9. Know that debt is not their friend.
10. Keep close to their customers
  - a. management knows the customers personally
  - b. the relationship with the customer determines success
  - c. the family does not let the internal customers (management) take precedence over those with the real money
  - d. any advertising should build confidence in the company first and the produce second (the long-term view)
  - e. believe that creating a customer is the ultimate purpose of a business
    - i by creating utility
    - ii by pricing
    - iii by adaption to the customer's social and economic needs.
    - iv by delivering what represent true value to the customer

11. Emphasize long-term planning
  - a. constantly monitor their market for trends and discontinuity
  - b. keep track of the competition
  - c. are alert to shifts in law and public policy
  - d. know their organizational strengths and weaknesses and capabilities
  - e. are aware of the economic situation.
  
12. Hold differing assumptions about:
  - a. the economic environment - success comes from adding value to the customer as opposed to through financial dealings
  - b. human nature - high trust as opposed to distrusting
  - c. time - future oriented - while honoring the past as opposed to near term oriented
  - d. humans are not a means to an end and should be developed
  - e. relationships - egalitarian and group oriented as opposed to individualistic.

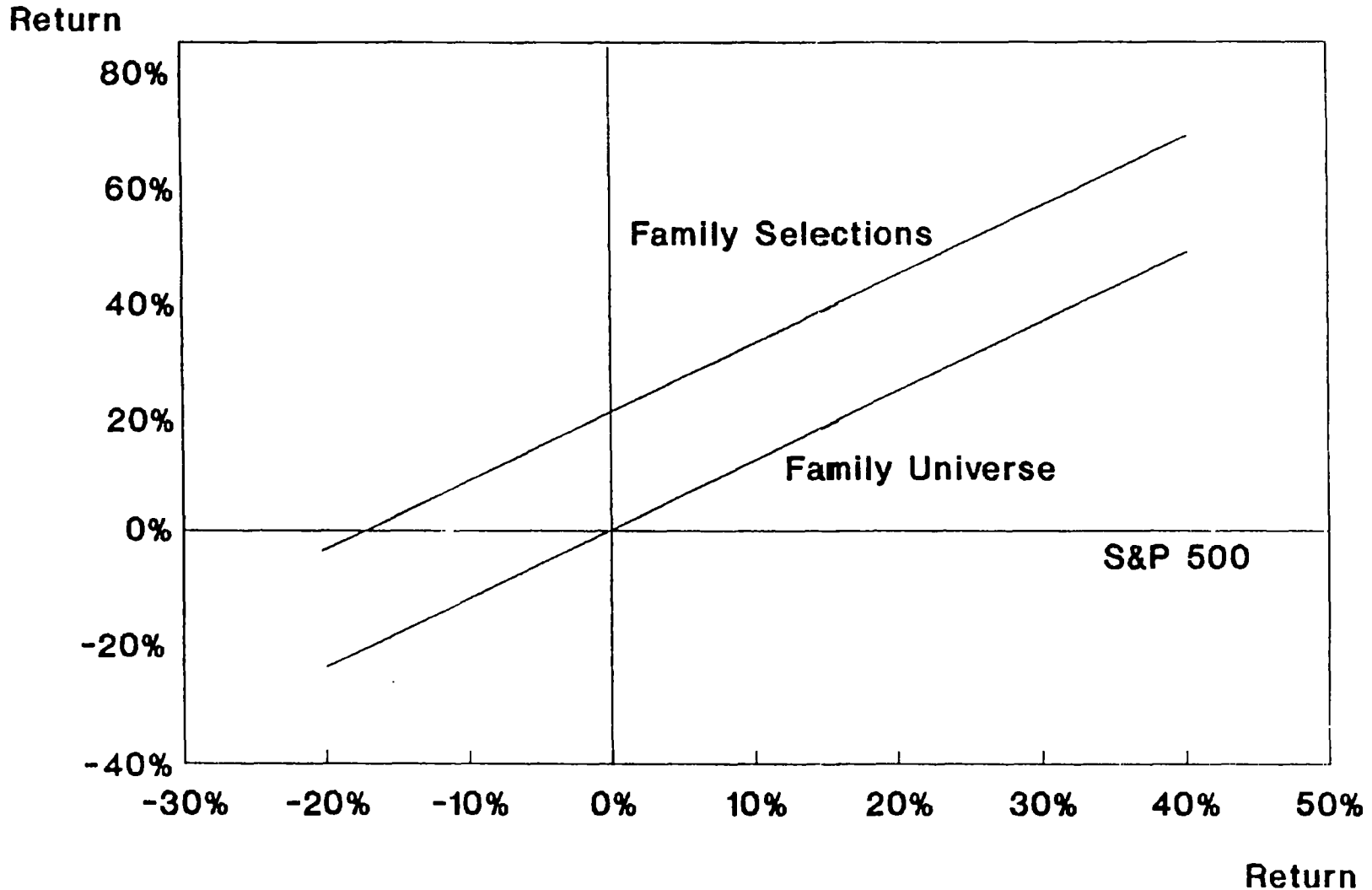
Reviewing annual reports, proxies and 10-Ks and interviewing respective management teams for these characteristics as well as their firm handle on valuing companies from a corporate finance perspective as opposed to investment perspective, (Return on Equity Profit Margin, Return on Assets, Asset Turnover, Inventory Turnover, Collection Period, Days Sales in Cash, Payables Period, Fixed Asset Turnover, Financial Leverage, Debt-to-Assets RATION, Debt-to-Equity Ratio, Times Interest Earned, Times Broken Covered, Current Ratio, Acid test, Return on Invested Capital, as opposed to Price/Earnings Ratio, Earnings Yield.

Relative Momentum) has yielded a group of good family controlled public companies with the following results:

# CUMULATIVE RETURNS: S&P 500, FAMILY UNIVERSE, FAMILY SELECTIONS

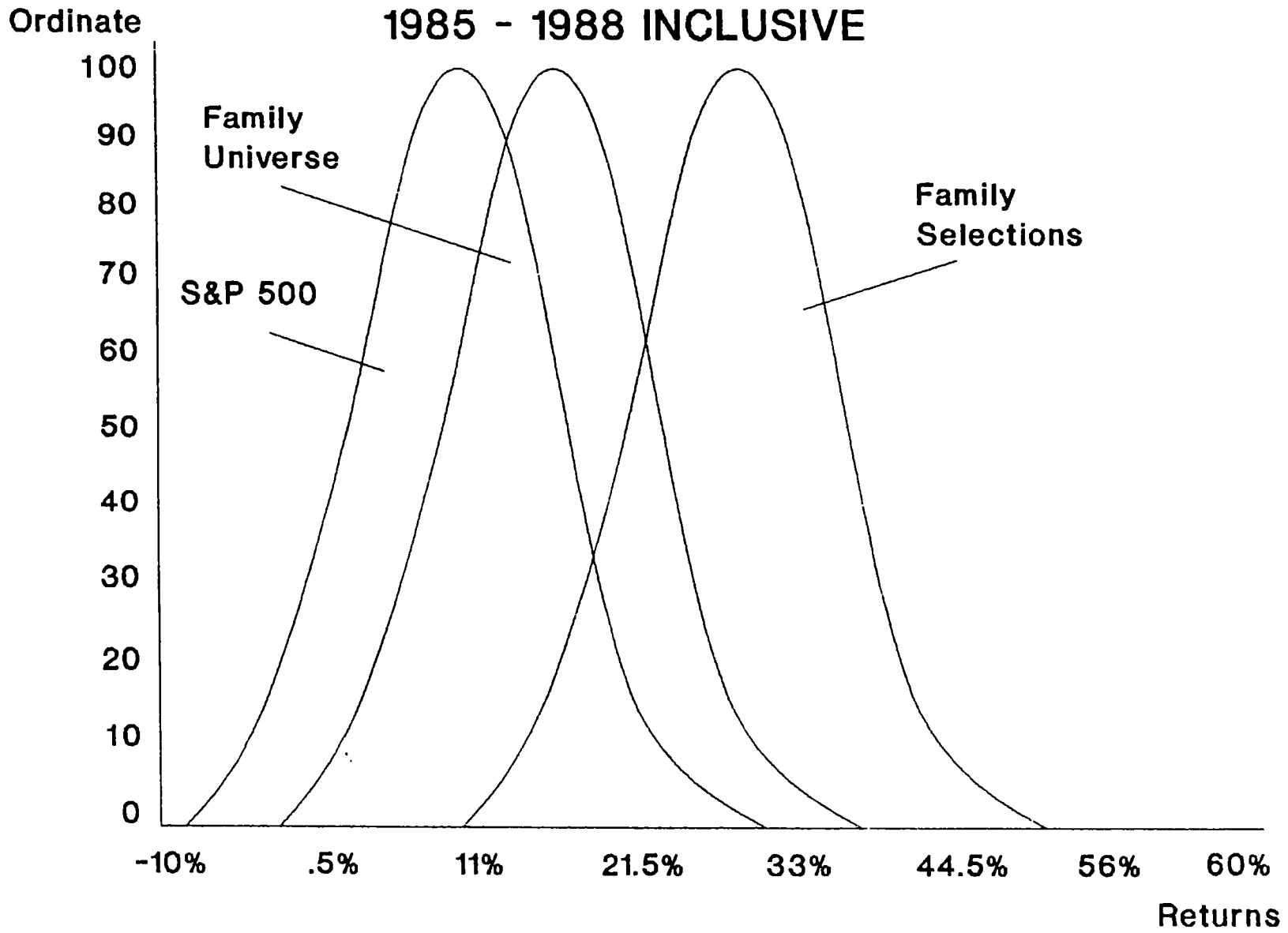


# LINEAR REGRESSION: S&P 500, FAMILY UNIVERSE, FAMILY SELECTIONS



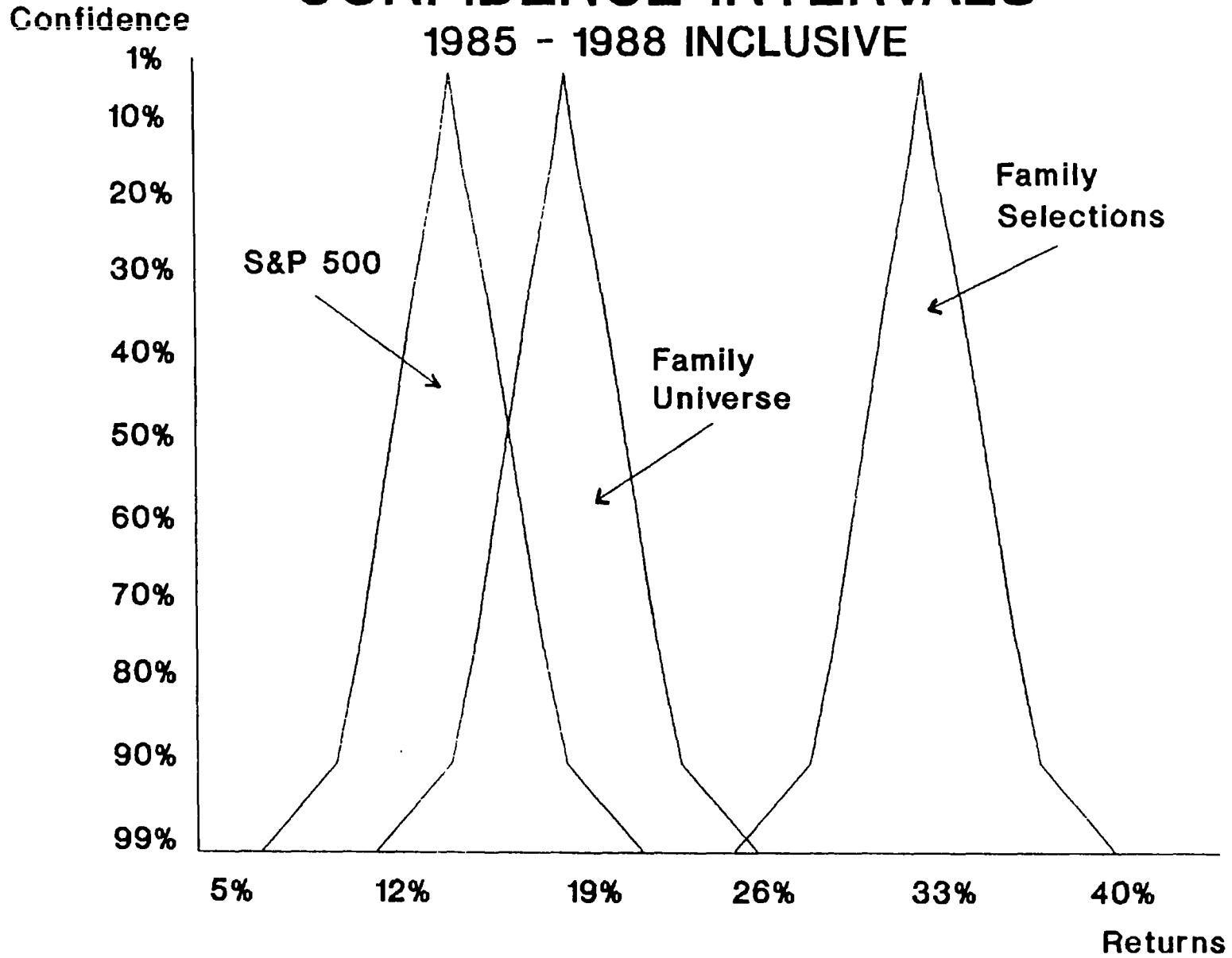
# NORMAL DISTRIBUTIONS

1985 - 1988 INCLUSIVE



# CONFIDENCE INTERVALS

1985 - 1988 INCLUSIVE



FAMILY SELECTIONS

ANNUAL RETURNS

1969	1970	1971	1972	1973	1974	1975	1976	1977	1978	1979	1980	1981	1982	1983	1984	1985	1986	1987	1988	YEAR
3.267913	7.462705	18.53334	21.15408	11.23910	6.515431	12.03838	13.84467	11.61880	11.28978	11.26051	12.66077	12.44341	15.45380	16.13087	15.74591	17.43167	18.38250	18.79353	19.27206	1969
	11.82789	26.99251	27.77981	13.32825	7.177088	13.57090	15.44160	12.70907	12.21871	12.09303	13.55591	13.46210	16.44869	17.10873	16.65098	18.37881	19.33753	19.72149	20.17996	1970
		44.21355	36.58981	13.83015	6.044930	13.92275	16.05513	12.83552	12.26766	12.12253	13.73017	13.61185	16.84227	17.52513	17.00334	18.82894	19.82329	20.20279	20.66184	1971
			29.36910	1.130684	-4.28400	7.401807	11.12097	8.314303	8.322527	8.649759	10.76863	10.93418	14.62790	15.53789	15.13650	17.19693	18.35236	18.84238	19.40290	1972
				-20.9439	-17.6693	8.942007	6.976258	4.533829	5.163933	5.974150	8.640077	9.035365	13.24951	14.35634	14.02364	16.30949	17.60233	18.17186	18.80614	1973
					-14.2591	14.06180	18.32361	12.09525	11.34048	11.27835	13.68735	13.53009	17.86392	18.65683	17.88400	20.11257	21.25054	21.81411	22.07661	1974
						51.73746	38.99952	22.57082	18.85555	17.23446	19.16066	18.17581	22.64646	23.01869	21.69748	23.85046	24.80300	24.92824	25.19668	1975
							27.33089	10.16261	9.562444	9.912212	13.53792	13.35340	18.97333	19.83423	18.75063	21.36071	22.60545	22.92058	23.35868	1976
								-4.69083	1.630874	4.652593	10.32977	10.74770	17.63474	18.79993	17.71957	20.71488	22.14266	22.52730	23.03331	1977
									8.371888	9.662376	15.84538	14.98329	22.69187	23.24324	21.32510	24.33376	25.55605	25.64427	25.92253	1978
										10.96823	19.77324	17.27554	26.55828	26.45395	23.62976	26.79838	27.88741	27.72588	27.82687	1979
											29.27690	20.56239	32.22733	30.65193	26.33041	29.64812	30.50644	29.99120	29.85150	1980
												12.43532	33.72770	31.11352	25.60434	29.72249	30.71250	30.09356	29.92351	1981
													39.05231	41.58584	30.32826	34.44462	34.71005	33.29524	32.63465	1982
														26.03747	17.97443	27.11922	29.23049	28.66771	28.67964	1983
															10.42721	27.66358	30.31271	29.33380	29.21469	1984
																47.59033	41.56051	36.32977	34.39120	1985
																	35.77703	31.02588	30.25922	1986
																		26.44099	27.58498	1987
																			28.73933	1988



FAMILY SELECTIONS - S&P500

DIFF  
ANNUAL  
RETURNS

1969	1970	1971	1972	1973	1974	1975	1976	1977	1978	1979	1980	1981	1982	1983	1984	1985	1986	1987	1988	YEAR
11.43275	9.852902	15.67539	14.55694	9.246178	9.794543	10.36947	9.697332	8.809677	8.114495	6.824097	6.213378	7.129989	8.948723	8.649085	8.392171	8.794525	9.223416	9.833959	9.950143	1969
	8.080335	18.13652	15.75238	8.622959	9.448296	10.16356	9.405489	8.439036	7.699962	6.305071	5.670017	6.720774	8.722524	8.412298	8.152406	8.594843	9.063146	9.721976	9.850534	1970
		29.99760	20.17784	8.806330	9.765459	10.58332	9.632723	8.490630	7.652109	6.105395	5.421475	6.594110	8.777749	8.438378	8.157200	8.630278	9.127602	9.824066	9.954747	1971
			10.71888	0.421665	4.766001	6.615645	6.192250	5.529572	5.010934	3.615190	3.097133	4.610826	7.105898	6.868036	6.692816	7.279750	7.887459	8.699089	8.898821	1972
				-6.42463	2.701918	5.491470	5.222407	4.651948	4.208621	2.752705	2.267475	4.029045	6.781228	6.551055	6.389780	7.036828	7.700015	8.573506	8.792215	1973
					11.56348	13.19843	10.48398	8.256682	6.969129	4.761041	3.939583	5.754914	8.766392	8.320829	7.971480	8.580836	9.203049	10.05277	10.19648	1974
						14.58751	8.972998	6.411375	5.182154	2.722730	2.008642	4.492441	8.159020	7.705784	7.376812	8.105810	8.837257	9.809039	9.983633	1975
							4.057809	3.261070	2.784900	0.450155	0.020995	3.170596	7.402042	6.994196	6.719393	7.563541	8.395078	9.469029	9.679999	1976
								2.604907	2.253904	-0.55862	-0.87123	3.011180	7.902776	7.376437	7.019482	7.924565	8.801077	9.928837	10.11962	1977
									1.841920	-2.42151	-2.30769	3.122273	9.196055	8.351285	7.783541	8.744278	9.654809	10.83532	10.96625	1978
										-6.95914	-4.65864	3.578865	11.25096	9.812427	8.875523	9.853258	10.75821	11.95821	11.99213	1979
											-2.01826	8.923799	17.78036	14.32967	12.20047	12.86590	13.49083	14.49069	14.24698	1980
												17.51046	26.87584	19.39268	15.40290	15.64438	15.92102	16.68867	16.14360	1981
													38.77487	20.38342	14.50662	15.00239	15.47180	15.47797	15.87135	1982
														3.902949	4.318416	7.954109	10.25064	12.53043	12.49196	1983
															4.661130	9.955983	12.36620	14.64925	14.18117	1984
																16.59297	17.00744	18.50830	16.91662	1985
																	17.35124	19.28673	16.97458	1986
																		21.01092	16.78661	1987
																			12.29931	1988

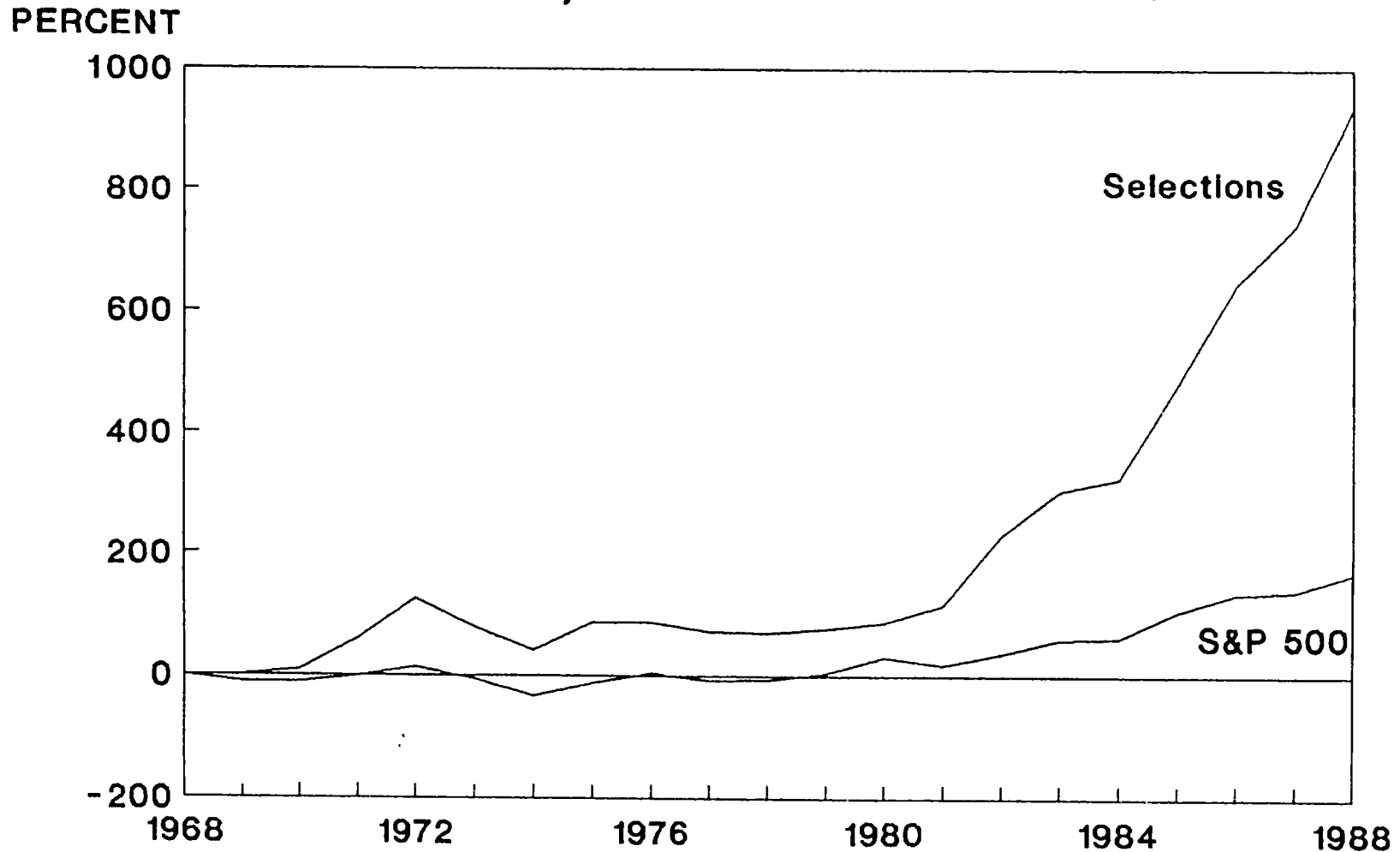
FAMILY SELECTIONS - FAMILY UNIVERSE

DIFF  
ANNUAL  
RETURNS

1969	1970	1971	1972	1973	1974	1975	1976	1977	1978	1979	1980	1981	1982	1983	1984	1985	1986	1987	1988	YEAR
10.68847	10.34776	11.24622	10.14971	8.290464	8.417970	7.408394	5.722308	3.742854	2.238858	1.215912	0.928500	1.080823	1.748721	1.090648	1.333969	1.649204	2.155539	2.573564	2.595021	1969
	9.955251	11.49726	9.852273	7.608685	7.937166	6.785195	4.895287	2.751305	1.165533	0.130128	-0.10277	0.151925	0.931549	0.269701	0.590918	0.966589	1.544878	2.023778	2.073584	1970
		13.27406	9.709546	6.798963	7.452487	6.126355	3.993049	1.671500	0.010210	-1.02147	-1.17964	-0.79972	0.108769	-0.55238	-0.14265	0.300195	0.956782	1.501119	1.581274	1971
			6.422212	4.363030	6.021053	4.721380	2.494337	0.142718	-1.49316	-2.44707	-2.48591	-1.94397	-0.89322	-1.52660	-1.01069	-0.49172	0.249948	0.866430	0.985476	1972
				2.893293	5.719283	4.245442	1.661066	-0.90299	-2.60383	-3.52574	-3.45793	-2.75622	-1.55416	-2.18756	-1.57418	-0.98413	-0.16143	0.520065	0.666204	1973
					8.678266	5.107316	0.994154	-2.27278	-4.17443	-5.05176	-4.77413	-3.77849	-2.29533	-2.95175	-2.18281	-1.47859	-0.52802	0.250570	0.428045	1974
						-2.30726	-5.77391	-7.88348	-8.96090	-9.08256	-8.10155	-6.39011	-4.37334	-4.91285	-3.81328	-2.88768	-1.70892	-0.75024	-0.48404	1975
							-8.72925	-9.88795	-10.5441	-10.2909	-8.95472	-6.87987	-4.59396	-5.16116	-3.93562	-2.92849	-1.66198	-0.64440	-0.36998	1976
								-10.6155	-11.2147	-10.6867	-8.98790	-6.54836	-3.96490	-4.69001	-3.39014	-2.33098	-1.00324	0.039676	0.280354	1977
									-11.8467	-10.6934	-8.30284	-5.34144	-2.31079	-3.44528	-2.12476	-1.03828	0.331602	1.364143	1.513107	1978
										-9.52498	-6.38770	-3.08459	0.330155	-1.56975	-0.36703	0.672318	2.022747	2.986093	2.990406	1979
											-2.81837	0.268738	4.027532	0.673189	1.620763	2.557971	3.855127	4.710169	4.522895	1980
												2.888918	7.432350	1.832724	2.675656	3.610378	4.946445	5.756869	5.415989	1981
													13.44720	1.142240	2.584922	3.813547	5.422559	6.306219	5.829162	1982
														-9.42760	-1.67732	1.129116	3.728251	5.105949	4.762429	1983
															4.742825	6.159456	7.965838	8.580705	7.486122	1984
																7.898446	9.921715	10.09040	8.284567	1985
																	11.72708	11.01916	8.380791	1986
																		10.34571	6.778017	1987
																			3.029445	1988

13

# CUMULATIVE RETURNS: S&P 500, FAMILY SELECTIONS



In Conclusion by merging ownership and control in the publicly held corporation and valuing the concern as an owner-entrepreneur, it is possible to achieve significantly superior long term returns to those companies whose institutional/investors and corporate managements have a tug-of-war over the near-term return streams of the economic engine that is the corporation.

## APPENDIX

Test of Past Correlations: S&P 500;  
Family Universe;  
Family Selections

S&P 500 Family Universe

$$r = 0.9759$$
$$r_{.990} = 0.735$$

$$0.9759 > 0.7350$$

Conclusion: 99% confidence of significant correlations between the S&P 500 and the Family Universe.

$$0.9759^2 = 0.9524$$

Conclusion: 95% of the aggregate return of the Family Universe is explained by the S&P 500.

S&P 500 Family Selections

$$r = 0.9456$$
$$r_{.990} = 0.735$$

$$0.9456 > 0.735$$

Conclusion: 99% confidence of significant correlation between the S&P 500 and The Family Selections.

$$0.9456^2 = 0.8942$$

Conclusion: 89% of the aggregate return of the Family Selections is explained by the S&P 500.

Test of Past Variances: S&P 500;  
 Family Universe;  
 Family Selections

	S&P 500	Family Universe
Intervals	10	10
Variances	44.6786	73.88795

$$x = 73.88795 \div 44.6786 = 1.6539$$

$$y = F_{.900} (9,9) = 2.44$$

$$2.44 > 1.6539$$

Conclusion: No significant difference between risk in S&P 500 and Family Universe.

$$1.60 (1.6539) > > 0.2481 (1.6539)$$

$$6.6652 \quad .4103$$

$$2.6462 > X > 1.0337$$

75% of the time the variances of the Family Universe divided by the variance of the S&P 500 will be within the region X.

	S&P 500	Family Selections
Intervals	10	10
Variances	44.67486	45.5174

$$x = 45.5174 \div 44.67486 = 1.0189$$

$$y = F_{.900} (9,9) = 2.44$$

$$2.44 > 1.0189$$

Conclusion: No significant difference between risk in S&P 500 and Family Selections.

$$1.600 (1.0189) = X > 0.6250 (1.0189)$$

$$1.6302 > X > .6368$$

75% of the time the variance of the Family Selections divided by the variance of the S&P 500 will be within the region X.

Test of Past Expected Returns: S&P 500;  
Family Universe: Family Selections

	S&P 500	Family Universe
Intervals	10	10
Average Returns	13.30665	18.36081
Variance	6.683925 <sup>2</sup>	8.59810 <sup>2</sup>

$$z_{.900} = 1.282$$

$$x = 1.282 \sqrt{\frac{6.683925^2 + 8.59810^2}{10}}$$

$$= 4.4150$$

$$z = 18.36081 - 13.330665$$

$$= 5.0542$$

$$5.0542 > 4.4150$$

Conclusion: 90% confidence the Family Universe has significantly outperformed the S&P 500 on a risk-adjusted-return basis.

90% confidence interval for the true difference between the returns of the Family Universe and the S&P 500:

$$9.4692 > > 0.6392$$

S&P 500      Family Selections

Time Intervals	10	10
Average Return	13.30665	31.55919
Variance	6.683925 <sup>2</sup>	6.746659 <sup>2</sup>

$$z_{.990} = 2.325$$

$$x = 2.325 \sqrt{\frac{6.683925^2 + 6.746659^2}{10}}$$

$$= 6.9824$$

$$31.55919 - 13.30665 = 18.2525$$

$$18.2525 > 6.9824$$

Conclusion: 99% confidence that the Family Selections, in the aggregate have significantly outperformed the S&P 500 on a risk-adjusted return basis.

99% confidence interval for the true difference between the aggregate return of the Family Selections and the S&P 500.





PROBABILITY FUNCTIONS

PERCENTAGE POINTS OF THE *t*-DISTRIBUTION—VALUES OF *t* IN TERMS OF *A* AND *v*

<i>A</i>	0.2	0.5	0.8	0.9	0.95	0.98	0.99	0.995	0.998	0.999	0.9999	0.99999	0.999999
1	0.325	1.000	3.078	6.314	12.706	31.821	63.657	127.321	318.309	636.619	6366.198	63661.977	636619.772
2	0.289	0.816	1.886	2.920	4.303	6.965	9.925	14.089	22.327	31.598	99.992	316.225	999.999
3	0.277	0.765	1.638	2.353	3.182	4.541	5.841	7.453	10.214	12.924	28.000	60.397	130.155
4	0.271	0.741	1.533	2.132	2.776	3.747	4.604	5.598	7.173	8.610	15.544	27.771	49.459
5	0.267	0.727	1.476	2.015	2.571	3.365	4.032	4.773	5.893	6.869	11.178	17.897	28.477
6	0.265	0.718	1.440	1.943	2.447	3.143	3.707	4.317	5.208	5.959	9.082	13.555	20.047
7	0.263	0.711	1.415	1.895	2.365	2.998	3.499	4.029	4.785	5.408	7.885	11.215	15.764
8	0.262	0.706	1.397	1.860	2.306	2.896	3.355	3.833	4.501	5.041	7.120	9.782	13.257
9	0.261	0.703	1.383	1.833	2.262	2.821	3.250	3.690	4.297	4.781	6.594	8.827	11.637
10	0.260	0.700	1.372	1.812	2.228	2.764	3.169	3.581	4.144	4.587	6.211	8.150	10.516
11	0.260	0.697	1.363	1.796	2.201	2.718	3.106	3.497	4.025	4.437	5.921	7.648	9.702
12	0.259	0.695	1.356	1.782	2.179	2.681	3.055	3.428	3.930	4.318	5.694	7.261	9.085
13	0.259	0.694	1.350	1.771	2.160	2.650	3.012	3.372	3.852	4.221	5.513	6.955	8.604
14	0.258	0.692	1.345	1.761	2.145	2.624	2.977	3.326	3.787	4.140	5.363	6.706	8.218
15	0.258	0.691	1.341	1.753	2.131	2.602	2.947	3.286	3.733	4.073	5.239	6.502	7.903
16	0.258	0.690	1.337	1.746	2.120	2.583	2.921	3.252	3.686	4.015	5.134	6.330	7.642
17	0.257	0.689	1.333	1.740	2.110	2.567	2.898	3.223	3.646	3.965	5.044	6.184	7.421
18	0.257	0.688	1.330	1.734	2.101	2.552	2.878	3.197	3.610	3.922	4.966	6.059	7.232
19	0.257	0.688	1.328	1.729	2.093	2.539	2.861	3.174	3.579	3.883	4.897	5.949	7.069
20	0.257	0.687	1.325	1.725	2.086	2.528	2.845	3.153	3.552	3.850	4.837	5.854	6.927
21	0.257	0.686	1.323	1.721	2.080	2.518	2.831	3.135	3.527	3.819	4.784	5.769	6.802
22	0.256	0.686	1.321	1.717	2.074	2.508	2.819	3.119	3.505	3.792	4.736	5.694	6.692
23	0.256	0.685	1.319	1.714	2.069	2.500	2.807	3.104	3.485	3.768	4.693	5.627	6.593
24	0.256	0.685	1.318	1.711	2.064	2.492	2.797	3.090	3.467	3.745	4.654	5.566	6.504
25	0.256	0.684	1.316	1.708	2.060	2.485	2.787	3.078	3.450	3.725	4.619	5.511	6.424
26	0.256	0.684	1.315	1.706	2.056	2.479	2.779	3.067	3.435	3.707	4.587	5.461	6.352
27	0.256	0.684	1.314	1.703	2.052	2.473	2.771	3.057	3.421	3.690	4.558	5.415	6.286
28	0.256	0.683	1.313	1.701	2.048	2.467	2.763	3.047	3.408	3.674	4.530	5.373	6.225
29	0.256	0.683	1.311	1.699	2.045	2.462	2.756	3.038	3.396	3.659	4.506	5.335	6.170
30	0.256	0.683	1.310	1.697	2.042	2.457	2.750	3.030	3.385	3.646	4.482	5.299	6.119
40	0.255	0.681	1.303	1.684	2.021	2.423	2.704	2.971	3.307	3.551	4.321	5.053	5.768
60	0.254	0.679	1.296	1.671	2.000	2.390	2.660	2.915	3.232	3.460	4.169	4.825	5.449
120	0.254	0.677	1.289	1.658	1.980	2.358	2.617	2.860	3.160	3.373	* 4.025	* 4.613	* 5.158
∞	0.253	0.674	1.282	1.645	1.960	2.326	2.576	2.807	3.090	3.291	* 3.891	* 4.417	* 4.892

$$A = A(t; v) = \left[ \sqrt{v} B \left( \frac{1}{2}, \frac{v}{2} \right) \right]^{-1} \int_{-t}^t \left( 1 + \frac{x^2}{v} \right)^{-\left( \frac{v+1}{2} \right)} dx$$

PROBABILITY FUNCTIONS

PERCENTAGE POINTS OF THE F-DISTRIBUTION—VALUES  
OF  $F$  IN TERMS OF  $Q, \nu_1, \nu_2$

$$Q(F; \nu_1, \nu_2) = 0.5$$

$\nu_2 \backslash \nu_1$	1	2	3	4	5	6	8	12	15	20	30	60	$\infty$
1	1.00	1.50	1.71	1.82	1.89	1.94	2.00	2.07	2.09	2.12	2.15	2.17	2.20
2	0.667	1.00	1.13	1.21	1.25	1.28	1.32	1.36	1.38	1.39	1.41	1.43	1.44
3	0.585	0.881	1.00	1.06	1.10	1.13	1.16	1.20	1.21	1.23	1.24	1.25	1.27
4	0.549	0.828	0.941	1.00	1.04	1.06	1.09	1.13	1.14	1.15	1.16	1.18	1.19
5	0.528	0.799	0.907	0.965	1.00	1.02	1.05	1.09	1.10	1.11	1.12	1.14	1.15
6	0.515	0.780	0.886	0.942	0.977	1.00	1.03	1.06	1.07	1.08	1.10	1.11	1.12
7	0.506	0.767	0.871	0.926	0.960	0.983	1.01	1.04	1.05	1.07	1.08	1.09	1.10
8	0.499	0.757	0.860	0.915	0.948	0.971	1.00	1.03	1.04	1.05	1.07	1.08	1.09
9	0.494	0.749	0.852	0.906	0.939	0.962	0.990	1.02	1.03	1.04	1.05	1.07	1.08
10	0.490	0.743	0.845	0.899	0.932	0.954	0.983	1.01	1.02	1.03	1.05	1.06	1.07
11	0.486	0.739	0.840	0.893	0.926	0.948	0.977	1.01	1.02	1.03	1.04	1.05	1.06
12	0.484	0.735	0.835	0.888	0.921	0.943	0.972	1.00	1.01	1.02	1.03	1.05	1.06
13	0.481	0.731	0.832	0.885	0.917	0.939	0.967	0.996	1.01	1.02	1.03	1.04	1.05
14	0.479	0.729	0.828	0.881	0.914	0.936	0.964	0.992	1.00	1.01	1.03	1.04	1.05
15	0.478	0.726	0.826	0.878	0.911	0.933	0.960	0.989	1.00	1.01	1.02	1.03	1.05
16	0.476	0.724	0.823	0.876	0.908	0.930	0.958	0.986	0.997	1.01	1.02	1.03	1.04
17	0.475	0.722	0.821	0.874	0.906	0.928	0.955	0.983	0.995	1.01	1.02	1.03	1.04
18	0.474	0.721	0.819	0.872	0.904	0.926	0.953	0.981	0.992	1.00	1.02	1.03	1.04
19	0.473	0.719	0.818	0.870	0.902	0.924	0.951	0.979	0.990	1.00	1.01	1.02	1.04
20	0.472	0.718	0.816	0.868	0.900	0.922	0.950	0.977	0.989	1.00	1.01	1.02	1.03
21	0.471	0.716	0.815	0.867	0.899	0.921	0.948	0.976	0.987	0.998	1.01	1.02	1.03
22	0.470	0.715	0.814	0.866	0.898	0.919	0.947	0.974	0.986	0.997	1.01	1.02	1.03
23	0.470	0.714	0.813	0.864	0.896	0.918	0.945	0.973	0.984	0.996	1.01	1.02	1.03
24	0.469	0.714	0.812	0.863	0.895	0.917	0.944	0.972	0.983	0.994	1.01	1.02	1.03
25	0.468	0.713	0.811	0.862	0.894	0.916	0.943	0.971	0.982	0.993	1.00	1.02	1.03
26	0.468	0.712	0.810	0.861	0.893	0.915	0.942	0.970	0.981	0.992	1.00	1.01	1.03
27	0.467	0.711	0.809	0.861	0.892	0.914	0.941	0.969	0.980	0.991	1.00	1.01	1.03
28	0.467	0.711	0.808	0.860	0.892	0.913	0.940	0.968	0.979	0.990	1.00	1.01	1.02
29	0.466	0.710	0.808	0.859	0.891	0.912	0.940	0.967	0.978	0.990	1.00	1.01	1.02
30	0.466	0.709	0.807	0.858	0.890	0.912	0.939	0.966	0.978	0.989	1.00	1.01	1.02
40	0.463	0.705	0.802	0.854	0.885	0.907	0.934	0.961	0.972	0.983	0.994	1.01	1.02
60	0.461	0.701	0.798	0.849	0.880	0.901	0.928	0.956	0.967	0.978	0.989	1.00	1.01
120	0.458	0.697	0.793	0.844	0.875	0.896	0.923	0.950	0.961	0.972	0.983	0.994	1.01
$\infty$	0.455	0.693	0.789	0.839	0.870	0.891	0.918	0.945	0.956	0.967	0.978	0.989	1.00

$$Q(F; \nu_1, \nu_2) = 0.25$$

$\nu_2 \backslash \nu_1$	1	2	3	4	5	6	8	12	15	20	30	60	$\infty$
1	5.83	7.50	8.20	8.58	8.82	8.98	9.19	9.41	9.49	9.58	9.67	9.76	9.85
2	2.57	3.00	3.15	3.23	3.28	3.31	3.35	3.39	3.41	3.43	3.44	3.46	3.48
3	1.92	2.28	2.36	2.39	2.41	2.42	2.44	2.45	2.46	2.46	2.47	2.47	2.47
4	1.81	2.00	2.05	2.06	2.07	2.08	2.08	2.08	2.08	2.08	2.08	2.08	2.08
5	1.69	1.85	1.88	1.89	1.89	1.89	1.89	1.89	1.89	1.88	1.88	1.87	1.87
6	1.62	1.76	1.78	1.79	1.79	1.78	1.78	1.77	1.76	1.76	1.75	1.74	1.74
7	1.57	1.70	1.72	1.72	1.71	1.71	1.70	1.68	1.68	1.67	1.66	1.65	1.65
8	1.54	1.66	1.67	1.66	1.66	1.65	1.64	1.62	1.62	1.61	1.60	1.59	1.58
9	1.51	1.62	1.63	1.63	1.62	1.61	1.60	1.58	1.57	1.56	1.55	1.54	1.53
10	1.49	1.60	1.60	1.59	1.59	1.58	1.56	1.54	1.53	1.52	1.51	1.50	1.48
11	1.47	1.58	1.58	1.57	1.56	1.55	1.53	1.51	1.50	1.49	1.48	1.47	1.45
12	1.46	1.56	1.56	1.55	1.54	1.53	1.51	1.49	1.48	1.47	1.45	1.44	1.42
13	1.45	1.55	1.55	1.53	1.52	1.51	1.49	1.47	1.46	1.45	1.43	1.42	1.40
14	1.44	1.53	1.53	1.52	1.51	1.50	1.48	1.45	1.44	1.43	1.41	1.40	1.38
15	1.43	1.52	1.52	1.51	1.49	1.48	1.46	1.44	1.43	1.41	1.40	1.38	1.36
16	1.42	1.51	1.51	1.50	1.48	1.47	1.45	1.43	1.41	1.40	1.38	1.36	1.34
17	1.42	1.51	1.50	1.49	1.47	1.46	1.44	1.41	1.40	1.39	1.37	1.35	1.33
18	1.41	1.50	1.49	1.48	1.46	1.45	1.43	1.40	1.39	1.38	1.36	1.34	1.32
19	1.41	1.49	1.49	1.47	1.46	1.44	1.42	1.40	1.38	1.37	1.35	1.33	1.30
20	1.40	1.49	1.48	1.47	1.45	1.44	1.42	1.39	1.37	1.36	1.34	1.32	1.29
21	1.40	1.48	1.48	1.46	1.44	1.43	1.41	1.38	1.37	1.35	1.33	1.31	1.28
22	1.40	1.48	1.47	1.45	1.44	1.42	1.40	1.37	1.36	1.34	1.32	1.30	1.28
23	1.39	1.47	1.47	1.45	1.43	1.42	1.40	1.37	1.35	1.34	1.32	1.30	1.27
24	1.39	1.47	1.46	1.44	1.43	1.41	1.39	1.36	1.35	1.33	1.31	1.29	1.26
25	1.39	1.47	1.46	1.44	1.42	1.41	1.39	1.36	1.34	1.33	1.31	1.28	1.25
26	1.38	1.46	1.45	1.44	1.42	1.41	1.38	1.35	1.34	1.32	1.30	1.28	1.25
27	1.38	1.46	1.45	1.43	1.42	1.40	1.38	1.35	1.33	1.32	1.30	1.27	1.24
28	1.38	1.46	1.45	1.43	1.41	1.40	1.38	1.34	1.33	1.31	1.29	1.27	1.24
29	1.38	1.45	1.45	1.43	1.41	1.40	1.37	1.34	1.32	1.31	1.29	1.26	1.23
30	1.38	1.45	1.44	1.42	1.41	1.39	1.37	1.34	1.32	1.30	1.28	1.26	1.23
40	1.36	1.44	1.42	1.40	1.39	1.37	1.35	1.31	1.30	1.28	1.25	1.22	1.19
60	1.35	1.42	1.41	1.38	1.37	1.35	1.32	1.29	1.27	1.25	1.22	1.19	1.15
120	1.31	1.40	1.39	1.37	1.35	1.33	1.30	1.26	1.24	1.22	1.19	1.16	1.10
$\infty$	1.32	1.39	1.37	1.35	1.33	1.31	1.28	1.24	1.22	1.19	1.16	1.12	1.00

PROBABILITY FUNCTIONS

PERCENTAGE POINTS OF THE F-DISTRIBUTION—VALUES  
OF  $F$  IN TERMS OF  $Q, \nu_1, \nu_2$

$Q(F|\nu_1, \nu_2) = 0.1$

$\nu_2 \backslash \nu_1$	1	2	3	4	5	6	8	12	15	20	30	60	$\infty$
1	39.36	49.50	53.59	55.83	57.24	58.20	59.44	60.71	61.22	61.74	62.26	62.79	63.33
2	8.53	5.00	4.16	3.74	3.29	3.33	3.37	3.41	3.42	3.44	3.46	3.47	3.49
3	5.54	5.46	5.39	5.34	5.31	5.28	5.25	5.22	5.20	5.18	5.17	5.15	5.13
4	4.54	4.32	4.19	4.11	4.05	4.01	3.95	3.90	3.87	3.84	3.82	3.79	3.76
5	4.06	3.78	3.62	3.52	3.45	3.40	3.34	3.27	3.24	3.21	3.17	3.14	3.10
6	3.78	3.46	3.29	3.18	3.11	3.05	2.98	2.90	2.87	2.84	2.80	2.76	2.72
7	3.59	3.26	3.07	2.96	2.88	2.83	2.75	2.67	2.63	2.59	2.56	2.51	2.47
8	3.46	3.11	2.92	2.81	2.73	2.67	2.59	2.50	2.46	2.42	2.38	2.34	2.29
9	3.36	3.01	2.81	2.69	2.61	2.55	2.47	2.38	2.34	2.30	2.25	2.21	2.16
10	3.29	2.92	2.73	2.61	2.52	2.46	2.38	2.28	2.24	2.20	2.16	2.11	2.06
11	3.23	2.86	2.66	2.54	2.45	2.39	2.30	2.21	2.17	2.12	2.08	2.03	1.97
12	3.18	2.81	2.61	2.48	2.39	2.33	2.24	2.15	2.10	2.06	2.01	1.96	1.90
13	3.14	2.76	2.56	2.43	2.35	2.28	2.20	2.10	2.05	2.01	1.96	1.90	1.85
14	3.10	2.73	2.52	2.39	2.31	2.24	2.15	2.05	2.01	1.96	1.91	1.86	1.80
15	3.07	2.70	2.49	2.36	2.27	2.21	2.12	2.02	1.97	1.92	1.87	1.82	1.76
16	3.05	2.67	2.46	2.33	2.24	2.18	2.09	1.99	1.94	1.89	1.84	1.78	1.72
17	3.03	2.64	2.44	2.31	2.22	2.15	2.06	1.96	1.91	1.86	1.81	1.75	1.69
18	3.01	2.62	2.42	2.29	2.20	2.13	2.04	1.93	1.89	1.84	1.78	1.72	1.66
19	2.99	2.61	2.40	2.27	2.18	2.11	2.02	1.91	1.86	1.81	1.76	1.70	1.63
20	2.97	2.59	2.38	2.25	2.16	2.09	2.00	1.89	1.84	1.79	1.74	1.68	1.61
21	2.96	2.57	2.36	2.23	2.14	2.08	1.98	1.87	1.83	1.78	1.72	1.66	1.59
22	2.95	2.56	2.35	2.22	2.13	2.06	1.97	1.86	1.81	1.76	1.70	1.64	1.57
23	2.94	2.55	2.34	2.21	2.11	2.05	1.95	1.84	1.80	1.74	1.69	1.62	1.55
24	2.93	2.54	2.33	2.19	2.10	2.04	1.94	1.83	1.78	1.73	1.67	1.61	1.53
25	2.92	2.53	2.32	2.18	2.09	2.02	1.93	1.82	1.77	1.72	1.66	1.59	1.52
26	2.91	2.52	2.31	2.17	2.08	2.01	1.92	1.81	1.76	1.71	1.65	1.58	1.50
27	2.90	2.51	2.30	2.17	2.07	2.00	1.91	1.80	1.75	1.70	1.64	1.57	1.49
28	2.89	2.50	2.29	2.16	2.06	2.00	1.90	1.79	1.74	1.69	1.63	1.56	1.48
29	2.89	2.50	2.28	2.15	2.06	1.99	1.89	1.78	1.73	1.68	1.62	1.55	1.47
30	2.88	2.49	2.28	2.14	2.05	1.98	1.88	1.77	1.72	1.67	1.61	1.54	1.46
40	2.84	2.44	2.23	2.09	2.00	1.93	1.83	1.71	1.66	1.61	1.54	1.47	1.38
60	2.79	2.39	2.18	2.04	1.95	1.87	1.77	1.66	1.60	1.54	1.48	1.40	1.29
120	2.75	2.35	2.13	1.99	1.90	1.82	1.72	1.60	1.55	1.48	1.41	1.32	1.19
$\infty$	2.71	2.30	2.08	1.94	1.85	1.77	1.67	1.55	1.49	1.42	1.34	1.24	1.00

$Q(F|\nu_1, \nu_2) = 0.05$

$\nu_2 \backslash \nu_1$	1	2	3	4	5	6	8	12	15	20	30	60	$\infty$
1	161.4	199.5	215.7	224.6	230.2	234.0	238.9	243.9	245.9	248.0	250.1	252.2	254.3
2	18.51	19.00	19.16	19.25	19.30	19.33	19.37	19.41	19.43	19.45	19.46	19.48	19.50
3	10.13	9.55	9.28	9.12	9.01	8.94	8.85	8.74	8.70	8.66	8.62	8.57	8.53
4	7.71	6.94	6.59	6.39	6.26	6.16	6.04	5.91	5.86	5.80	5.75	5.69	5.63
5	6.61	5.79	5.41	5.19	5.05	4.95	4.82	4.68	4.62	4.56	4.50	4.43	4.36
6	5.99	5.14	4.76	4.53	4.39	4.28	4.15	4.00	3.94	3.87	3.81	3.74	3.67
7	5.59	4.74	4.35	4.12	3.97	3.87	3.73	3.57	3.51	3.44	3.38	3.30	3.23
8	5.32	4.46	4.07	3.84	3.69	3.58	3.44	3.28	3.22	3.15	3.08	3.01	2.93
9	5.12	4.26	3.86	3.63	3.48	3.37	3.23	3.07	3.01	2.94	2.86	2.79	2.71
10	4.96	4.10	3.71	3.48	3.33	3.22	3.07	2.91	2.85	2.77	2.70	2.62	2.54
11	4.84	3.98	3.59	3.36	3.20	3.09	2.95	2.79	2.72	2.65	2.57	2.49	2.40
12	4.75	3.89	3.49	3.26	3.11	3.00	2.85	2.69	2.62	2.54	2.47	2.38	2.30
13	4.67	3.81	3.41	3.18	3.03	2.92	2.77	2.60	2.53	2.46	2.38	2.30	2.21
14	4.60	3.74	3.34	3.11	2.96	2.85	2.70	2.53	2.46	2.39	2.31	2.22	2.13
15	4.54	3.68	3.29	3.06	2.90	2.79	2.64	2.48	2.40	2.33	2.25	2.16	2.07
16	4.49	3.63	3.24	3.01	2.85	2.74	2.59	2.42	2.35	2.28	2.19	2.11	2.01
17	4.45	3.59	3.20	2.96	2.81	2.70	2.55	2.38	2.31	2.23	2.15	2.06	1.96
18	4.41	3.55	3.16	2.93	2.77	2.66	2.51	2.34	2.27	2.19	2.11	2.02	1.92
19	4.38	3.52	3.13	2.90	2.74	2.63	2.48	2.31	2.23	2.16	2.07	1.98	1.88
20	4.35	3.49	3.10	2.87	2.71	2.60	2.45	2.28	2.20	2.12	2.04	1.95	1.84
21	4.32	3.47	3.07	2.84	2.68	2.57	2.42	2.25	2.18	2.10	2.01	1.92	1.81
22	4.30	3.44	3.05	2.82	2.66	2.55	2.40	2.23	2.15	2.07	1.98	1.89	1.78
23	4.28	3.42	3.03	2.80	2.64	2.53	2.37	2.20	2.13	2.05	1.96	1.86	1.76
24	4.26	3.40	3.01	2.78	2.62	2.51	2.36	2.18	2.11	2.03	1.94	1.84	1.73
25	4.24	3.39	2.99	2.76	2.60	2.49	2.34	2.16	2.09	2.01	1.92	1.82	1.71
26	4.23	3.37	2.98	2.74	2.59	2.47	2.32	2.15	2.07	1.99	1.90	1.80	1.69
27	4.21	3.35	2.96	2.73	2.57	2.46	2.31	2.13	2.06	1.97	1.88	1.79	1.67
28	4.20	3.34	2.95	2.71	2.56	2.45	2.29	2.12	2.04	1.96	1.87	1.77	1.65
29	4.18	3.33	2.93	2.70	2.55	2.43	2.28	2.10	2.03	1.94	1.85	1.75	1.64
30	4.17	3.32	2.92	2.69	2.53	2.42	2.27	2.09	2.01	1.93	1.84	1.74	1.62
40	4.08	3.23	2.84	2.61	2.45	2.34	2.18	2.00	1.92	1.84	1.74	1.64	1.51
60	4.00	3.15	2.76	2.53	2.37	2.25	2.10	1.92	1.84	1.75	1.65	1.53	1.39
120	3.92	3.07	2.68	2.45	2.29	2.17	2.02	1.83	1.75	1.66	1.55	1.43	1.25
$\infty$	3.84	3.00	2.60	2.37	2.21	2.10	1.94	1.75	1.67	1.57	1.46	1.32	1.00

PROBABILITY FUNCTIONS

PERCENTAGE POINTS OF THE F-DISTRIBUTION—VALUES  
OF F IN TERMS OF Q,  $\nu_1, \nu_2$   
 $Q(F|\nu_1, \nu_2) = 0.025$

$\nu_2 \backslash \nu_1$	1	2	3	4	5	6	8	12	15	20	30	60	$\infty$
1	647.8	799.5	864.2	899.6	921.8	937.1	956.7	976.7	984.9	993.1	1001	1010	1018
2	38.51	39.00	39.17	39.25	39.30	39.33	39.37	39.41	39.43	39.45	39.46	39.48	39.50
3	17.44	16.04	15.44	15.10	14.88	14.73	14.54	14.34	14.25	14.17	14.08	13.99	13.90
4	12.22	10.65	9.98	9.60	9.36	9.20	8.98	8.75	8.66	8.56	8.46	8.36	8.26
5	10.01	8.43	7.76	7.39	7.15	6.98	6.76	6.52	6.43	6.33	6.23	6.12	6.02
6	8.81	7.26	6.60	6.23	5.99	5.82	5.60	5.37	5.27	5.17	5.07	4.96	4.85
7	8.07	6.54	5.89	5.52	5.29	5.12	4.90	4.67	4.57	4.47	4.36	4.25	4.14
8	7.57	6.06	5.42	5.05	4.82	4.65	4.43	4.20	4.10	4.00	3.89	3.78	3.67
9	7.21	5.71	5.08	4.72	4.48	4.32	4.10	3.87	3.77	3.67	3.56	3.45	3.33
10	6.94	5.46	4.83	4.47	4.24	4.07	3.85	3.62	3.52	3.42	3.31	3.20	3.08
11	6.72	5.26	4.63	4.28	4.04	3.88	3.66	3.43	3.33	3.23	3.12	3.00	2.88
12	6.55	5.10	4.47	4.12	3.89	3.73	3.51	3.28	3.18	3.07	2.96	2.85	2.72
13	6.41	4.97	4.35	4.00	3.77	3.60	3.39	3.15	3.05	2.95	2.84	2.72	2.60
14	6.30	4.86	4.24	3.89	3.66	3.50	3.29	3.05	2.95	2.84	2.73	2.61	2.49
15	6.20	4.77	4.15	3.80	3.58	3.41	3.20	2.96	2.86	2.76	2.64	2.52	2.40
16	6.12	4.69	4.08	3.73	3.50	3.34	3.12	2.89	2.79	2.68	2.57	2.45	2.32
17	6.04	4.62	4.01	3.66	3.44	3.28	3.06	2.82	2.72	2.62	2.50	2.38	2.25
18	5.98	4.56	3.95	3.61	3.38	3.22	3.01	2.77	2.67	2.56	2.44	2.32	2.19
19	5.92	4.51	3.90	3.56	3.33	3.17	2.96	2.72	2.62	2.51	2.39	2.27	2.13
20	5.87	4.46	3.86	3.51	3.29	3.13	2.91	2.68	2.57	2.46	2.35	2.22	2.09
21	5.83	4.42	3.82	3.48	3.25	3.09	2.87	2.64	2.53	2.42	2.31	2.18	2.04
22	5.79	4.38	3.78	3.44	3.22	3.05	2.84	2.60	2.50	2.39	2.27	2.14	2.00
23	5.75	4.35	3.75	3.41	3.18	3.02	2.81	2.57	2.47	2.36	2.24	2.11	1.97
24	5.72	4.32	3.72	3.38	3.15	2.99	2.78	2.54	2.44	2.33	2.21	2.08	1.94
25	5.69	4.29	3.69	3.35	3.13	2.97	2.75	2.51	2.41	2.30	2.18	2.05	1.91
26	5.66	4.27	3.67	3.33	3.10	2.94	2.73	2.49	2.39	2.28	2.16	2.03	1.88
27	5.63	4.24	3.65	3.31	3.08	2.92	2.71	2.47	2.36	2.25	2.13	2.00	1.85
28	5.61	4.22	3.63	3.29	3.06	2.90	2.69	2.45	2.34	2.23	2.11	1.98	1.83
29	5.59	4.20	3.61	3.27	3.04	2.88	2.67	2.43	2.32	2.21	2.09	1.96	1.81
30	5.57	4.18	3.59	3.25	3.03	2.87	2.65	2.41	2.31	2.20	2.07	1.94	1.79
40	5.42	4.05	3.46	3.13	2.90	2.74	2.53	2.29	2.18	2.07	1.94	1.80	1.64
60	5.29	3.93	3.34	3.01	2.79	2.63	2.41	2.17	2.06	1.94	1.82	1.67	1.48
120	5.15	3.80	3.23	2.89	2.67	2.52	2.30	2.05	1.94	1.82	1.69	1.53	1.31
$\infty$	5.02	3.69	3.12	2.79	2.57	2.41	2.19	1.94	1.83	1.71	1.57	1.39	1.00

$Q(F|\nu_1, \nu_2) = 0.01$

$\nu_2 \backslash \nu_1$	1	2	3	4	5	6	8	12	15	20	30	60	$\infty$
1	4052	4999.5	5403	5625	5764	5859	5982	6106	6157	6209	6261	6313	6366
2	98.50	99.00	99.17	99.25	99.30	99.33	99.37	99.42	99.43	99.45	99.47	99.48	99.50
3	34.12	30.82	29.46	28.71	28.24	27.91	27.49	27.05	26.87	26.69	26.50	26.32	26.13
4	21.20	18.00	16.69	15.98	15.52	15.21	14.80	14.37	14.20	14.02	13.84	13.65	13.46
5	16.26	13.27	12.06	11.39	10.97	10.67	10.29	9.89	9.72	9.55	9.38	9.20	9.02
6	13.75	10.92	9.78	9.15	8.75	8.47	8.10	7.72	7.56	7.40	7.23	7.06	6.88
7	12.25	9.55	8.45	7.85	7.46	7.19	6.84	6.47	6.31	6.16	5.99	5.82	5.65
8	11.26	8.65	7.59	7.01	6.63	6.37	6.03	5.67	5.52	5.36	5.20	5.03	4.86
9	10.56	8.02	6.99	6.42	6.06	5.80	5.47	5.11	4.96	4.81	4.65	4.48	4.31
10	10.04	7.56	6.55	5.99	5.64	5.39	5.06	4.71	4.56	4.41	4.25	4.08	3.91
11	9.65	7.21	6.22	5.67	5.32	5.07	4.74	4.40	4.25	4.10	3.94	3.78	3.60
12	9.33	6.93	5.95	5.41	5.06	4.82	4.50	4.16	4.01	3.86	3.70	3.54	3.36
13	9.07	6.70	5.74	5.21	4.86	4.62	4.30	3.96	3.82	3.66	3.51	3.34	3.17
14	8.86	6.51	5.56	5.04	4.69	4.46	4.14	3.80	3.66	3.51	3.35	3.18	3.00
15	8.68	6.36	5.42	4.89	4.56	4.32	4.00	3.67	3.52	3.37	3.21	3.05	2.87
16	8.53	6.23	5.29	4.77	4.44	4.20	3.89	3.55	3.41	3.26	3.10	2.93	2.75
17	8.40	6.11	5.18	4.67	4.34	4.10	3.79	3.46	3.31	3.16	3.00	2.83	2.65
18	8.29	6.01	5.09	4.58	4.25	4.01	3.71	3.37	3.23	3.08	2.92	2.75	2.57
19	8.18	5.93	5.01	4.50	4.17	3.94	3.63	3.30	3.15	3.00	2.84	2.67	2.49
20	8.10	5.85	4.94	4.43	4.10	3.87	3.56	3.23	3.09	2.94	2.78	2.61	2.42
21	8.02	5.78	4.87	4.37	4.04	3.81	3.51	3.17	3.03	2.88	2.72	2.55	2.36
22	7.95	5.72	4.82	4.31	3.99	3.76	3.45	3.12	2.98	2.83	2.67	2.50	2.31
23	7.88	5.66	4.76	4.26	3.94	3.71	3.41	3.07	2.93	2.78	2.62	2.45	2.26
24	7.82	5.61	4.72	4.22	3.90	3.67	3.36	3.03	2.89	2.74	2.58	2.40	2.21
25	7.77	5.57	4.68	4.18	3.85	3.63	3.32	2.99	2.85	2.70	2.54	2.36	2.17
26	7.72	5.53	4.64	4.14	3.82	3.59	3.29	2.96	2.81	2.66	2.50	2.33	2.13
27	7.68	5.49	4.60	4.11	3.78	3.56	3.26	2.93	2.78	2.63	2.47	2.29	2.10
28	7.64	5.45	4.57	4.07	3.75	3.53	3.23	2.90	2.75	2.60	2.44	2.26	2.06
29	7.60	5.42	4.54	4.04	3.73	3.50	3.20	2.87	2.73	2.57	2.41	2.23	2.03
30	7.56	5.39	4.51	4.02	3.70	3.47	3.17	2.84	2.70	2.55	2.39	2.21	2.01
40	7.31	5.18	4.31	3.83	3.51	3.29	2.99	2.66	2.52	2.37	2.20	2.02	1.80
60	7.08	4.98	4.13	3.65	3.34	3.12	2.82	2.50	2.35	2.20	2.03	1.84	1.60
120	6.85	4.79	3.95	3.48	3.17	2.96	2.66	2.34	2.19	2.03	1.86	1.66	1.38
$\infty$	6.63	4.61	3.78	3.32	3.02	2.80	2.51	2.18	2.04	1.88	1.70	1.47	1.00

PROBABILITY FUNCTIONS

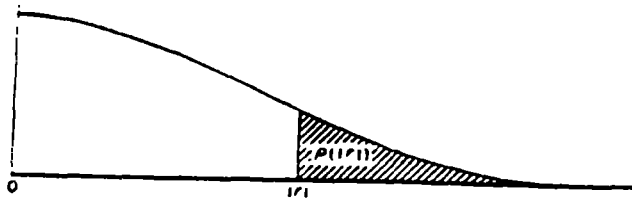
PERCENTAGE POINTS OF THE F-DISTRIBUTION—VALUES

OF F IN TERMS OF  $Q, \nu_1, \nu_2$   
 $Q(F|\nu_1, \nu_2) = 0.005$

$\nu_2 \backslash \nu_1$	1	2	3	4	5	6	8	12	15	20	30	60	$\infty$
1	16211	20000	21615	22500	23056	23437	23925	24426	24630	24836	25044	25253	25465
2	198.5	199.0	199.2	199.2	199.3	199.3	199.4	199.4	199.4	199.4	199.5	199.5	199.5
3	55.55	49.80	47.47	46.19	45.35	44.84	44.13	43.39	43.08	42.78	42.47	42.15	41.83
4	31.33	26.20	24.26	23.15	22.46	21.97	21.35	20.70	20.44	20.17	19.89	19.61	19.32
5	22.78	18.31	16.53	15.56	14.94	14.51	13.96	13.38	13.15	12.90	12.66	12.40	12.14
6	18.63	14.54	12.92	12.03	11.46	11.07	10.57	10.03	9.81	9.59	9.36	9.12	8.88
7	16.24	12.40	10.88	10.05	9.52	9.16	8.68	8.18	7.97	7.75	7.53	7.31	7.08
8	14.69	11.04	9.60	8.81	8.30	7.95	7.50	7.01	6.81	6.61	6.40	6.18	5.95
9	13.61	10.11	8.72	7.96	7.47	7.13	6.69	6.23	6.03	5.83	5.62	5.41	5.19
10	12.83	9.43	8.08	7.34	6.87	6.54	6.12	5.66	5.47	5.27	5.07	4.86	4.64
11	12.23	8.91	7.60	6.88	6.42	6.10	5.68	5.24	5.05	4.86	4.65	4.44	4.23
12	11.75	8.51	7.23	6.52	6.07	5.76	5.35	4.91	4.72	4.53	4.33	4.12	3.90
13	11.37	8.19	6.93	6.23	5.79	5.48	5.08	4.64	4.46	4.27	4.07	3.87	3.65
14	11.06	7.92	6.68	6.00	5.56	5.26	4.86	4.43	4.25	4.06	3.86	3.66	3.44
15	10.80	7.70	6.48	5.80	5.37	5.07	4.67	4.25	4.07	3.88	3.69	3.48	3.26
16	10.58	7.51	6.30	5.64	5.21	4.91	4.52	4.10	3.92	3.73	3.54	3.33	3.11
17	10.38	7.35	6.16	5.50	5.07	4.78	4.39	3.97	3.79	3.61	3.41	3.21	2.98
18	10.22	7.21	6.03	5.37	4.96	4.66	4.28	3.86	3.68	3.50	3.30	3.10	2.87
19	10.07	7.09	5.92	5.27	4.85	4.56	4.18	3.76	3.59	3.40	3.21	3.00	2.78
20	9.94	6.99	5.82	5.17	4.76	4.47	4.09	3.68	3.50	3.32	3.12	2.92	2.69
21	9.83	6.89	5.73	5.09	4.68	4.39	4.01	3.60	3.43	3.24	3.05	2.84	2.61
22	9.73	6.81	5.65	5.02	4.61	4.32	3.94	3.54	3.36	3.18	2.98	2.77	2.55
23	9.63	6.73	5.58	4.95	4.54	4.26	3.88	3.47	3.30	3.12	2.92	2.71	2.48
24	9.55	6.66	5.52	4.89	4.49	4.21	3.83	3.42	3.25	3.06	2.87	2.66	2.43
25	9.48	6.60	5.46	4.84	4.43	4.15	3.78	3.37	3.20	3.01	2.82	2.61	2.38
26	9.41	6.54	5.41	4.79	4.38	4.10	3.73	3.33	3.15	2.97	2.77	2.56	2.33
27	9.34	6.49	5.36	4.74	4.34	4.06	3.69	3.28	3.11	2.93	2.73	2.52	2.29
28	9.28	6.44	5.32	4.70	4.30	4.02	3.65	3.25	3.07	2.89	2.69	2.48	2.25
29	9.23	6.40	5.28	4.66	4.26	3.98	3.61	3.21	3.04	2.86	2.66	2.45	2.21
30	9.18	6.35	5.24	4.62	4.23	3.95	3.58	3.18	3.01	2.82	2.63	2.42	2.18
40	8.83	6.07	4.98	4.37	3.99	3.71	3.35	2.95	2.78	2.60	2.40	2.18	1.93
60	8.49	5.79	4.73	4.14	3.76	3.49	3.13	2.74	2.57	2.39	2.19	1.96	1.69
120	8.18	5.54	4.50	3.92	3.55	3.28	2.93	2.54	2.37	2.19	1.98	1.75	1.43
$\infty$	7.88	5.30	4.28	3.72	3.35	3.09	2.74	2.36	2.19	2.00	1.79	1.55	1.00

$Q(F|\nu_1, \nu_2) = 0.001$

$\nu_2 \backslash \nu_1$	1	2	3	4	5	6	8	12	15	20	30	60	$\infty$
1	(5) 4.053	(5) 5.000	(5) 5.404	(5) 5.625	(5) 5.764	(5) 5.859	(5) 5.981	(5) 6.107	(5) 6.158	(5) 6.209	(5) 6.261	(5) 6.313	(5) 6.366
2	998.5	999.0	999.2	999.2	999.3	999.3	999.4	999.4	999.4	999.4	999.5	999.5	999.5
3	167.0	148.5	141.1	137.1	134.6	132.8	130.6	128.3	127.4	126.4	125.4	124.5	123.5
4	74.14	61.25	56.18	53.44	51.71	50.53	49.00	47.41	46.76	46.10	45.43	44.75	44.05
5	47.18	37.12	33.20	31.09	29.75	28.84	27.64	26.42	25.91	25.39	24.87	24.33	23.79
6	35.51	27.00	23.70	21.92	20.81	20.03	19.03	17.99	17.56	17.12	16.67	16.21	15.75
7	29.25	21.69	18.77	17.19	16.21	15.52	14.63	13.71	13.32	12.93	12.53	12.12	11.70
8	25.42	18.49	15.83	14.39	13.49	12.86	12.04	11.19	10.84	10.48	10.11	9.73	9.33
9	22.86	16.39	13.90	12.56	11.71	11.13	10.37	9.57	9.24	8.90	8.55	8.19	7.81
10	21.04	14.91	12.55	11.28	10.48	9.92	9.20	8.45	8.13	7.80	7.47	7.12	6.76
11	19.69	13.81	11.56	10.35	9.58	9.05	8.35	7.63	7.32	7.01	6.68	6.35	6.00
12	18.64	12.97	10.80	9.63	8.89	8.38	7.71	7.00	6.71	6.40	6.09	5.76	5.42
13	17.81	12.31	10.21	9.07	8.35	7.86	7.21	6.52	6.23	5.93	5.63	5.30	4.97
14	17.14	11.78	9.73	8.62	7.92	7.43	6.80	6.13	5.85	5.56	5.25	4.94	4.60
15	16.59	11.34	9.34	8.25	7.57	7.09	6.47	5.81	5.54	5.25	4.95	4.64	4.31
16	16.12	10.97	9.00	7.94	7.27	6.81	6.19	5.55	5.27	4.99	4.70	4.39	4.06
17	15.72	10.66	8.73	7.68	7.02	6.56	5.96	5.32	5.05	4.78	4.48	4.18	3.85
18	15.38	10.39	8.49	7.46	6.81	6.35	5.76	5.13	4.87	4.59	4.30	4.00	3.67
19	15.08	10.16	8.28	7.26	6.62	6.18	5.59	4.97	4.70	4.43	4.14	3.84	3.51
20	14.82	9.95	8.10	7.10	6.46	6.02	5.44	4.82	4.56	4.29	4.00	3.70	3.38
21	14.59	9.77	7.94	6.95	6.32	5.88	5.31	4.70	4.44	4.17	3.88	3.58	3.26
22	14.38	9.61	7.80	6.81	6.19	5.76	5.19	4.58	4.33	4.06	3.78	3.48	3.15
23	14.19	9.47	7.67	6.69	6.08	5.65	5.09	4.48	4.23	3.96	3.68	3.38	3.05
24	14.03	9.34	7.55	6.59	5.98	5.55	4.99	4.39	4.14	3.87	3.59	3.29	2.97
25	13.88	9.22	7.45	6.49	5.88	5.46	4.91	4.31	4.06	3.79	3.52	3.22	2.89
26	13.74	9.12	7.36	6.41	5.80	5.38	4.83	4.24	3.99	3.72	3.44	3.15	2.82
27	13.61	9.02	7.27	6.33	5.73	5.31	4.76	4.17	3.92	3.66	3.38	3.08	2.75
28	13.50	8.93	7.19	6.25	5.66	5.24	4.69	4.11	3.86	3.60	3.32	3.02	2.69
29	13.39	8.85	7.12	6.19	5.59	5.18	4.64	4.05	3.80	3.54	3.27	2.97	2.64
30	13.29	8.77	7.05	6.12	5.53	5.12	4.58	4.00	3.75	3.49	3.22	2.92	2.59
40	12.61	8.25	6.60	5.70	5.13	4.73	4.21	3.64	3.40	3.15	2.87	2.57	2.23
60	11.97	7.76	6.17	5.31	4.76	4.37	3.87	3.31	3.08	2.83	2.55	2.25	1.89
120	11.38	7.32	5.79	4.95	4.42	4.04	3.55	3.02	2.78	2.53	2.26	1.95	1.54
$\infty$	10.83	6.91	5.42	4.62	4.10	3.74	3.27	2.74	2.51	2.27	1.99	1.66	1.00



**CRITICAL ABSOLUTE VALUES OF CORRELATION  
COEFFICIENT  $r^*$**

5% points and 1% points (in boldface) for equal-tails test of hypothesis  $\rho = 0$ .

$f$	Total number of variables				$f$	Total number of variables			
	2	3	4	5		2	3	4	5
1	.997	.999	.999	.999	24	.388	.470	.523	.562
	<b>1.000</b>	<b>1.000</b>	<b>1.000</b>	<b>1.000</b>		<b>.496</b>	<b>.565</b>	<b>.609</b>	<b>.642</b>
2	.950	.975	.983	.987	25	.381	.462	.514	.553
	<b>.990</b>	<b>.995</b>	<b>.997</b>	<b>.998</b>		<b>.487</b>	<b>.555</b>	<b>.600</b>	<b>.633</b>
3	.878	.930	.950	.961	26	.374	.454	.506	.545
	<b>.959</b>	<b>.976</b>	<b>.983</b>	<b>.987</b>		<b>.478</b>	<b>.546</b>	<b>.590</b>	<b>.624</b>
4	.811	.881	.912	.930	27	.367	.446	.498	.536
	<b>.817</b>	<b>.849</b>	<b>.862</b>	<b>.876</b>		<b>.470</b>	<b>.538</b>	<b>.582</b>	<b>.615</b>
5	.754	.836	.874	.898	28	.361	.439	.490	.529
	<b>.874</b>	<b>.917</b>	<b>.937</b>	<b>.949</b>		<b>.463</b>	<b>.530</b>	<b>.573</b>	<b>.606</b>
6	.707	.795	.839	.867	29	.355	.432	.482	.521
	<b>.834</b>	<b>.886</b>	<b>.911</b>	<b>.927</b>		<b>.456</b>	<b>.522</b>	<b>.565</b>	<b>.598</b>
7	.666	.758	.807	.838	30	.349	.426	.476	.514
	<b>.798</b>	<b>.855</b>	<b>.885</b>	<b>.904</b>		<b>.449</b>	<b>.514</b>	<b>.558</b>	<b>.591</b>
8	.632	.726	.777	.811	35	.325	.397	.445	.482
	<b>.765</b>	<b>.827</b>	<b>.860</b>	<b>.882</b>		<b>.418</b>	<b>.481</b>	<b>.523</b>	<b>.556</b>
9	.602	.697	.750	.786	40	.304	.373	.419	.455
	<b>.735</b>	<b>.800</b>	<b>.836</b>	<b>.861</b>		<b>.393</b>	<b>.454</b>	<b>.494</b>	<b>.526</b>
10	.576	.671	.726	.763	45	.288	.353	.397	.432
	<b>.708</b>	<b>.776</b>	<b>.814</b>	<b>.840</b>		<b>.372</b>	<b>.430</b>	<b>.470</b>	<b>.501</b>
11	.553	.648	.703	.741	50	.273	.336	.379	.412
	<b>.684</b>	<b>.753</b>	<b>.793</b>	<b>.821</b>		<b>.354</b>	<b>.410</b>	<b>.449</b>	<b>.479</b>
12	.532	.627	.683	.722	60	.250	.308	.348	.380
	<b>.661</b>	<b>.732</b>	<b>.773</b>	<b>.802</b>		<b>.325</b>	<b>.377</b>	<b>.414</b>	<b>.442</b>
13	.514	.608	.664	.703	70	.232	.286	.324	.354
	<b>.641</b>	<b>.712</b>	<b>.755</b>	<b>.785</b>		<b>.302</b>	<b>.351</b>	<b>.386</b>	<b>.413</b>
14	.497	.590	.646	.686	80	.217	.269	.304	.332
	<b>.623</b>	<b>.694</b>	<b>.737</b>	<b>.768</b>		<b>.283</b>	<b>.330</b>	<b>.362</b>	<b>.389</b>
15	.482	.574	.630	.670	90	.205	.254	.288	.315
	<b>.606</b>	<b>.677</b>	<b>.721</b>	<b>.752</b>		<b>.267</b>	<b>.312</b>	<b>.343</b>	<b>.368</b>
16	.468	.559	.615	.655	100	.195	.241	.274	.300
	<b>.590</b>	<b>.662</b>	<b>.706</b>	<b>.738</b>		<b>.254</b>	<b>.297</b>	<b>.327</b>	<b>.351</b>
17	.456	.545	.601	.641	125	.174	.216	.246	.269
	<b>.575</b>	<b>.647</b>	<b>.691</b>	<b>.724</b>		<b>.228</b>	<b>.266</b>	<b>.294</b>	<b>.316</b>
18	.444	.532	.587	.628	150	.159	.198	.225	.247
	<b>.561</b>	<b>.633</b>	<b>.678</b>	<b>.710</b>		<b>.208</b>	<b>.244</b>	<b>.270</b>	<b>.290</b>
19	.433	.520	.575	.615	200	.138	.172	.196	.215
	<b>.549</b>	<b>.620</b>	<b>.665</b>	<b>.698</b>		<b>.181</b>	<b>.212</b>	<b>.234</b>	<b>.253</b>
20	.423	.509	.563	.604	300	.113	.141	.160	.176
	<b>.537</b>	<b>.608</b>	<b>.652</b>	<b>.685</b>		<b>.148</b>	<b>.174</b>	<b>.192</b>	<b>.208</b>
21	.413	.498	.552	.592	400	.098	.122	.139	.153
	<b>.526</b>	<b>.596</b>	<b>.641</b>	<b>.674</b>		<b>.128</b>	<b>.151</b>	<b>.167</b>	<b>.180</b>
22	.404	.488	.542	.582	500	.088	.109	.124	.137
	<b>.515</b>	<b>.585</b>	<b>.630</b>	<b>.663</b>		<b>.115</b>	<b>.135</b>	<b>.150</b>	<b>.162</b>
23	.396	.479	.532	.572	1000	.062	.077	.088	.097
	<b>.505</b>	<b>.574</b>	<b>.619</b>	<b>.652</b>		<b>.081</b>	<b>.096</b>	<b>.106</b>	<b>.115</b>